4 Performance Relationships for Specific Systems

In this chapter, automated transit systems are classified and studied according to the geometry of the lines. There are four classifications: shuttle, loop, line haul, and network. All transit systems are composed of one or more of these types. Hence, the performance of any transit system may be studied using the relationships developed.

4.1 Shuttle Systems

Simple Shuttle

A simple shuttle is diagrammed in figure 4-1. Only one vehicle can be used, and it follows the velocity profile of figure 2-3 in moving from one station to the other. The distance D_s between stations is measured to the center of the stopped vehicle. For a given V_L, the minimum possible value of D_s is the value of $\hat{D}$ in figure 2-4 for the case $t_D = 0$, or twice the stopping distance given by equation (2.2.6).

The travel time from one station to the other counting the dwell time t_D at either station is derived in section 2.4 and is given as t_s by equation (2.4.3). Because V_L appears in the denominator in the second term and in the numerator in the third, a value of V_L exists that minimizes t_s. By differentiation, it is seen that t_s is minimum if $V_L^2 = D_s a_m$. But, discounting jerk, $D_s = V_L^2/a_m$ is twice the stopping distance. Thus, t_s is minimized if the vehicle accelerates to the midpoint between stations, then decelerates to a stop. As D_s increases, the corresponding value of V_L to minimize t_s quickly becomes to large to be practical, and minimum t_s cannot be attained.

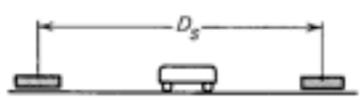


Figure 4-1. A Simple Shuttle

Three useful characteristic times for shuttles can be derived from t_s:

T_1: The time to wait for a vehicle called from the other station

T_2: The average wait time if the vehicle continuously shuttles back and forth and waits t_D seconds at each end

T_3: The effective time headway

These quantities are given by the following equations:

$$T_1 = t_s - t_D = \frac{D_s}{V_L} + \frac{V_L}{a_m} + 1 \text{ second} \tag{4.1.1}$$

$$T_2 = T_1 + t_D = t_s \tag{4.1.2}$$

$$T_3 = 2T_2 \tag{4.1.3}$$

in which we have assumed $a_m/J_1 = 1$ second because this value gives the maximum value of J_1 permissible from the standpoint of comfort.

The capacity of a shuttle in terms of the effective number of vehicles per hour passing a fixed point in one direction is

$$\text{Capacity} = \frac{3600}{T_3} = \frac{1800 V_L}{D_s + V_L(t_D + \frac{V_L}{a_m} + 1)} \tag{4.1.4}$$

Equations (4.1.1) through (4.1.4) are plotted in figure 4-2. The upper right-hand quadrant is a plot of equation (4.1.1) for $a_m = 1.25$ m/s². This value is appropriate for standing-passenger vehicles. It is used for shuttles because the round-trip time for a shuttle is long enough so that it is necessary to accommodate all the people who wish service on a particular trip, but short enough that provision for seating is unnecessary. Also, with no seats the vehicle can be used for transporting beds, food carts, and other objects as well as people. The lines of T_1 versus D_s terminate on the left end at the minimum value of D_s possible for the given value of V_L. The envelope of the end points of these lines is a parabola.

The upper left-hand quadrant is a plot of equation (4.1.2) for several practical values of t_D. The lower left-hand quadrant is a plot of equation (4.1.3), and the lower right-hand quadrant shows equation (4.1.4). Plotted in this manner, all of the necessary performance characteristics of a shuttle can be understood from a single chart. The dashed lines in figure 4-2 provide an example for the case $D_s = 600$ m, $V_L = 10$ m/s, and $t_D = 20$ s. Enter the chart at $D_s = 0.6$ km and move up to the curve corresponding to $V_L = 10$

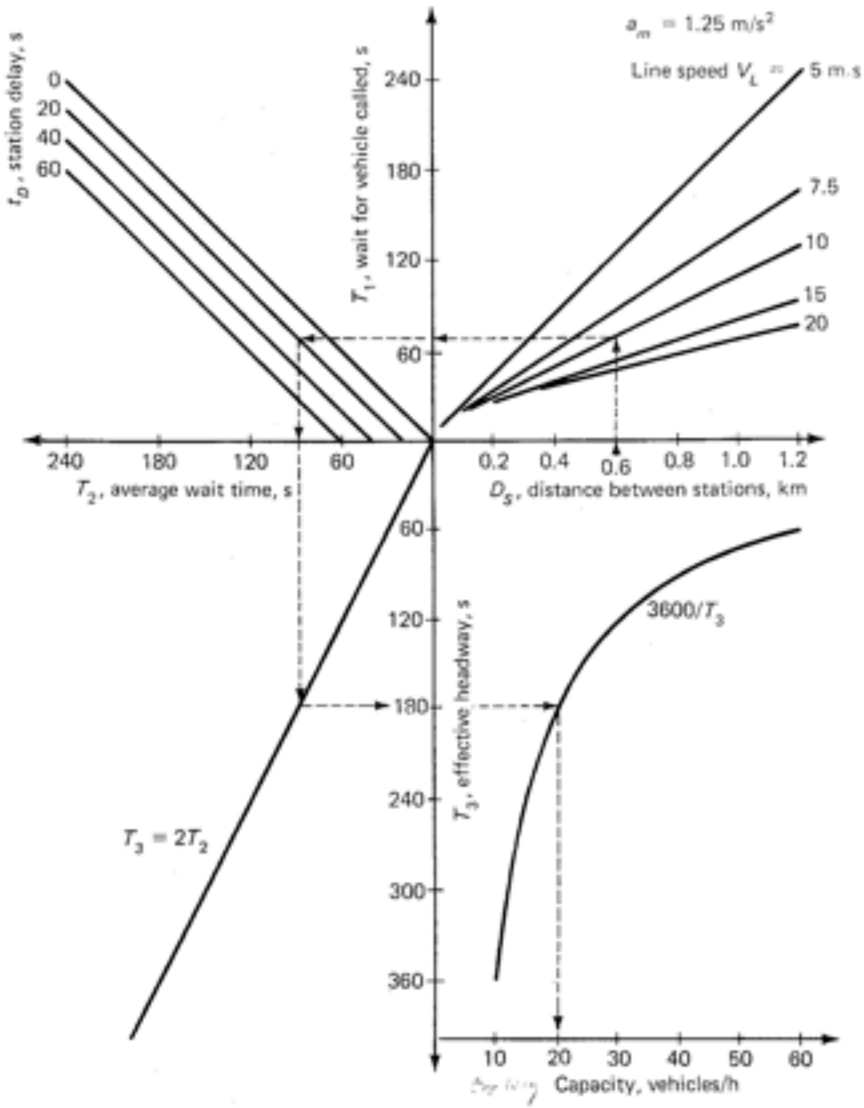


Figure 4-2. Characteristic Times for and Capacity of Simple Shuttles

m/s. Move left to the T_1 ordinate and read $T_1 = 69$ s. Continue left to the diagonal line corresponding to $t_D = 20$ s and turn 90 degrees down to the T_2 axis and read $T_2 = 89$ s. Continue down to the diagonal line labeled $T_3 = 2T_2$ and turn right to the T_3 ordinate. Read $T_3 = 178$ s and continue right to the hyperbola labeled $3600/T_3$. Turn down to the capacity abscissa and read 20.2 vehicles per hour.

Simple Shuttle With Intermediate Stations

Consider the configuration shown in figure 4-3. In the time and capacity relationships derived, we will assume there are n stations separated by arbitrary distances D_{s_i}, each greater than the minimum possible value shown in figure 4-2. Such a configuration is exactly analogous to an elevator and can be referred to as a "horizontal elevator." It would operate in an on-demand mode exactly as an elevator.

The time characteristics can be found from equations (4.1.1 through 4.1.3) and figure 4-2. The time to wait for the vehicle called from any other station nonstop is the value of T_1 in figure 4-2 corresponding to the distance D_s from which the vehicle is called. If, however, the vehicle makes m intermediate stops each with station delay t_D, the wait time is the sum of the values of T_1 corresponding to the $m + 1$ station spacings between m intermediate stops plus mt_D. If the D_{s_i} are all the same, the wait time is

$$T_{1m} = mt_D + (m + 1)\left(\frac{D_s}{V_L} + \frac{V_L}{a_m} + 1\right) \tag{4.1.5}$$

The characteristic times T_2 and T_3 have meaning only if the vehicle continues to shuttle back and forth with an average station delay t_D. Then, for an n-station system

$$\begin{aligned} T_{2_n} &= \sum_{i=1}^{n-1}\left(t_D + \frac{D_{s_i}}{V_L} + \frac{V_L}{a_m} + 1\right) \\ &= (n - 1)\left(t_D + \frac{D_{s_{av}}}{V_L} + \frac{V_L}{a_m} + 1\right) \\ &= (n - 1)T_2 \end{aligned} \tag{4.1.6}$$

in which $D_{s_{av}}$ is the average station spacing, and T_2 is found from figure 4-2 corresponding to $D_{s_{av}}$. The effective time headway $T_3 = 2T_{2_n}$ as in equation (4.1.3). The effective capacity is the value given in figure 4-2 corresponding to $D_{s_{av}}$ divided by $n - 1$.

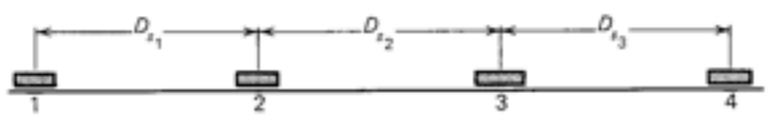


Figure 4-3. Simple Shuttle with Intermediate Stations

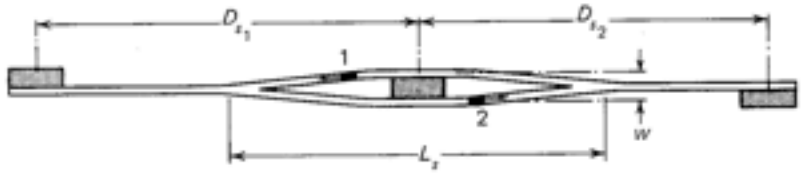


Figure 4-4. A Two-Vehicle Shuttle

Two-Vehicle Shuttle

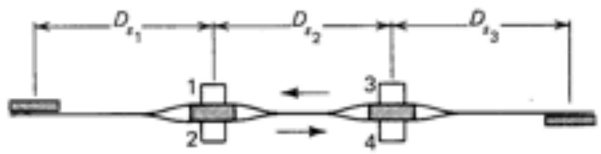
The major advantage of a simple shuttle is that only one guideway is required. Its major disadvantage is that with only one guideway, only one vehicle can be used, thus limiting capacity. Greater capacity without the expense of a double guideway the entire length can be obtained by using a double guideway around one or more intermediate stations. The Ford Motor Company has used such a configuration in their installation at Bradley Field, Hartford, Connecticut and at the Fairlane Shopping Center in Dearborn, Michigan.

The configuration is in general as shown in figure 4-4, in which it is assumed that D_{s_1} and D_{s_2} may differ, and the middle station uses a central platform. The length of curved guideway is minimized if it is designed according to the theory of section 3.5.

The characteristic times for a two-vehicle shuttle are each one-half the values for a three-station simple shuttle, given by equations (4.1.5) and (4.1.6). The capacity is double that of the three-station shuttle.

Four-Vehicle Shuttle

Consider the concept of figure 4-4 with two intermediate stations, diagrammed in figure 4-5. At time zero the four vehicles are at the two central stations, with vehicles 1 and 3 headed left, and vehicles 2 and 4 headed right. The vehicles advance to new positions in the time interval t_s, where t_s is given by equation (4.1.2) with D_s equal to the largest of the three station spacings shown in figure 4-5. In the first time interval ($t = 0$ to $t = t_s$), vehicle 1 can move to the left end station, and vehicle 4 to the right end station. Only one of vehicles 2 and 3 can move. In the table, the convention is adopted that the vehicle with the lowest number takes priority. Thus, in the first move vehicle 3 waits while vehicle 2 passes the middle segment. In each time step, the vehicle which must wait is encircled. We see that eight time steps are needed to bring the vehicles back to their original positions, and that each of the four vehicles waits out one time step twice.



Time				
0		1 2	3 4	
t_s	1		3 2	4
$2t_s$		3 1	4 2	
$3t_s$	3		4 1	2
$4t_s$		4 3	2 1	
$5t_s$	4	2 3		1
$6t_s$		2 4	1 3	
$7t_s$	2	1 4		3
$8t_s$		1 2	3 4	

Figure 4-5. Motions of a Four-Vehicle Shuttle

We see that the period of motion is $8t_s$ and that four vehicles pass a given station in a given direction in $8t_s$ seconds. Therefore, the effective time headway between vehicles is $T_3 = 2t_s$, exactly the same as for the simple shuttle (see equation (4.1.3)). Consequently, the capacity is also the same, and is given by figure 4-2.

A similar analysis assuming more than two intermediate stations shows that the capacity remains the same as for a simple shuttle. Only with the configuration of figure 4-4 is the capacity increased over that of a simple shuttle, and in that case by a factor of two. The advantage of shuttle configurations with more than one intermediate station is not to increase capacity, but to keep the capacity constant while increasing the length of the line. Without the intermediate stations, figure 4-2 or equation (4.1.4) shows that the capacity drops rapidly with D_s. As an example, consider a simple shuttle in which $V_L = 10$ m/s, $a_m = 1.25$ m/s^2, and $t_D = 10$ s. Then if D_s is doubled from 300 m to 600 m, the capacity drops from 36.7 vehicles per hour to 22.8 vehicles per hour, or by a factor of 1.6.

4.2 Station Throughput

The capacity of each of the systems discussed in the remainder of this chapter is limited by the number of vehicles or trains per hour that can move through a station, that is, the station throughput. In this section the term vehicle in general refers to either a vehicle or a train. For the purpose of this section, stations can be divided into two types: (1) The common type in which the vehicle flow through the station is unidirectional; and (2) the end-of-the-line station in which the vehicles leave by backing up and then switching to a second line.

Unidirectional Flow Station

Analysis of station throughput is aided by consideration of the distance-time diagrams of two successive vehicles as they pass through a station. Figure 4-6 shows such a diagram for a unidirectional station. The two velocity profiles are assumed to be identical in shape and are as defined in chapter 2. The line velocity, V_L, is the slope of the distance-time line before deceleration begins. The length of each vehicle or train is L, the station delay of vehicle 1 is t_D, and the two vehicles are assumed to be separated in time by an interval T. The problem is to determine how the minimum permissible time headway is related to other essential parameters.

Attention is focused on the trailing time line of vehicle 1 in figure 4-6, and on the leading time line of vehicle 2. Consider the trailing time line of vehicle 1 in a reference frame (x, t) with the origin in time at the moment vehicle 1 begins to leave the station. Then, in a reference frame (x', t') in which time and position move backwards and in which the origin in time is at the moment vehicle 2 stops in the station, the position-time diagram of vehicle 2 is identical to that of vehicle 1. To find it in reference frame x, t, we need only find the position-time diagram of vehicle 1 in $x - t$ coordinates and transform it by means of the equations

$$\left.\begin{aligned} x' &= L - x \\ t' &= T - t_D - t \end{aligned}\right\} \qquad (4.2.1)$$

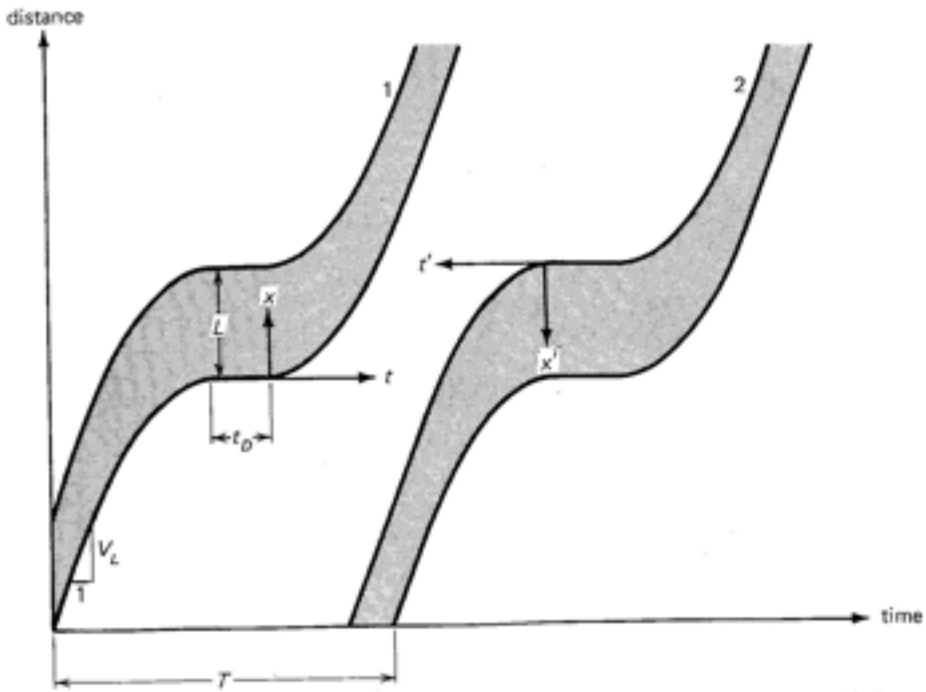


Figure 4-6. The Distance-Time Diagrams of Two Successive Vehicles Entering and Leaving a Station

Generally T will be an order of magnitude or more longer than terms in the position-time equations dependent on jerk. Therefore, in this analysis, jerk will be neglected. Then, if a is the acceleration, the position-time line of vehicle 1 is

$$x_1 = \frac{at^2}{2} \tag{4.2.2}$$

and

$$\dot{x}_1 = at \tag{4.2.3}$$

for $0 \leq t \leq V_L/a$. For $t > V_L/a$

$$x_1 = V_Lt - \frac{V_L^2}{2a} \tag{4.2.4}$$

Therefore, the position-time line of vehicle 2 is

$$\begin{aligned} x_2' &= \frac{at'^2}{2} & 0 \leq t' \leq V_L/a \\ &= V_Lt' - \frac{V_L^2}{2a} & t' > V_L/a \end{aligned}$$

Substituting these equations into equations (4.2.1),

$$\left.\begin{aligned} x_2 &= L - a/2(T - t_D - t)^2 && 0 \leqslant t' \leqslant V_L/a \\ &= L - V_L(T - t_D - t) + V_L^2/2a && t' > V_L/a \end{aligned}\right\} \quad (4.2.5)$$

If $T - t_D$ is greater than or equal to $2V_L/a$, the closest separation between x_1 and x_2 at a given t occurs when the velocities of both vehicles are V_L. Thus, $x_1 - x_2$ is found by subtracting the second of equations (4.2.5) from equation (4.2.4) to give

$$\Delta x_{\min} = V_L(T - t_D) - V_L^2/a - L \qquad T - t_D \geqslant 2V_L/a \quad (4.2.6)$$

The velocities of both vehicles are equal to V_L when $x_1 - x_2 = \Delta x_{\min}$.

If $T - t_D$ is less than $2V_L/a$, the closest separation occurs at velocities less than V_L. Thus, $\Delta x_{\min}$ is found by subtracting the first of equations (4.2.5) from equation (4.2.2), and

$$\Delta x_{\min} = at^2/2 - L + a/2(T - t_D - t)^2$$

In equation (4.2.6), t does not appear; however, in the present case $\Delta x_{\min}$ is a function of time. The minimum value occurs when

$$\frac{d\Delta x}{dt} = 0 = at - a(T - t_D - t)$$

that is, when

$$t = \frac{T - t_D}{2}$$

Thus

$$\Delta x_{\min} = \frac{a(T - t_D)^2}{4} - L \quad T - t_D < 2V_L/a \qquad (4.2.7)$$

The minimum permissible separation between two transit vehicles is dealt with in detail in chapter 7; however, for analysis of station flows it is adequate to let

$$\Delta x_{\min} = \frac{kV_{\min}^2}{2a_e} \qquad (4.2.8)$$

in which $V_{\min}$ is the velocity of the trailing vehicle at minimum separation, and a_e is the emergency deceleration rate. Thus, from equations (2.2.6), $V_{\min}^2/2a_e$ is the stopping distance of the trailing vehicle at minimum spacing if jerk and control time delay are neglected. The constant k, called the safety factor, is the ratio of minimum separation to stopping distance. The available stopping distance is less than $V_{\min}^2/2a_e$ because jerk and control delay have been neglected, but more because the lead vehicle cannot stop instantly.

If equation (4.2.8) is substituted into equation (4.2.7), in which at $\Delta x_{\min}$

$$V_{\min} = \frac{a(T - t_D)}{2}$$

and the result is solved for $T - t_D$, we have

$$T - t_D = 2\sqrt{\frac{L}{a(1 - ka/2a_e)}} \tag{4.2.9}$$

if $T - t_D \leqslant 2V_L/a$. If $T - t_D \geqslant 2V_L/a$, $V_{\min} = V_L$. Then substitute equation (4.2.8) into equation (4.2.6) to give

$$T - t_D = \frac{L}{V_L} + \frac{V_L}{a}\left(1 + \frac{ka}{2a_e}\right) \tag{4.2.10}$$

When $T - t_D = 2\,V_L/a$, equations (4.2.9) and (4.2.10) give the same result:

$$L = \frac{V_L^2}{a}\left(1 - \frac{ka}{2a_e}\right) \tag{4.2.11}$$

If L *is greater than the value given by equation (4.2.11), equation (4.2.10) holds; and if less, equation (4.2.9) holds.*

For train systems, k is generally taken equal to at least two, and a_e is chosen only slightly greater than a. Therefore the dimensionless parameter $ka/2a_e$ is approximately equal to 1 and equation (4.2.10) holds for trains of all lengths. If equation (4.2.10) is differentiated with respect to V_L and the result is set equal to zero, it is seen that T reaches a minimum value when

$$V_L = \left(\frac{aL}{1 + \alpha}\right)^{1/2} \tag{4.2.12}$$

in which

$$\alpha = \frac{ka}{2a_e} \tag{4.2.13}$$

Substituting equation (4.2.12) into equation (4.2.10),

$$T_{\min} = t_D + 2\left[\frac{(1+\alpha)L}{a}\right]^{1/2} \qquad (4.2.14)$$

For a ten-car train of 20-m cars, $L = 200$ m, and with $a = 1.25$ m/s², $t_D = 15$ s, $\alpha = 1$, $T_{\min} = 50.8$ s, and the corresponding value of V_L is 11.2 m/s. This is generally felt to be too low a line speed to give an adequate average speed. If $V_L = 25$ m/s with the same values of the other parameters the headway increases, from equation (4.2.10), to 63 s.

In order to present the results graphically, introduce, in addition to equation (4.2.13), the dimensionless parameters

$$\bar{T} = (T - t_D)\frac{a}{V_L} \qquad (4.2.15)$$

$$\bar{L} = \frac{La}{V_L^2} \qquad (4.2.16)$$

Then, equations (4.2.9) and (4.2.10) become

$$\left.\begin{aligned} \bar{T} &= 2\sqrt{\frac{\bar{L}}{1-\alpha}} & \bar{T} &\leq 2 \\ &= \bar{L} + 1 + \alpha & \bar{T} &\geq 2 \end{aligned}\right\} \qquad (4.2.17)$$

Note from equation (4.2.9) that at shorter headways, the headway into a station is independent of line speed. Equations (4.2.17) are plotted as the solid lines in figure 4-7 for a family of values of α. Typically a_e lies in the range from a to $2a$ and k ranges from one to two. Therefore, the curves corresponding to $\alpha = 1/4$ to 1 are in the practical range. If α is greater than 1, the second of equations (4.2.17) holds for the whole range of positive values of $\bar{L}$. Using equations (4.2.12) and (4.2.16), one can see that as a function of V_L, the second of equations (4.2.17) reaches a minimum when $\bar{L} = 1 + \alpha$. The dashed line in figure 4-7 connects these minimum points.

End-of-the-Line Station

Consider a station at the end of a transit line, which to save space and conserve on track length is arranged so that a vehicle or train entering it must back up onto a parallel line to continue in the opposite direction. A diagram of the station and the corresponding position-time diagrams are shown in figure 4-8. As before, L is the length of the train, t_D is the station dwell time, V_L is the line speed, and a is the deceleration rate. In the present case, the position-time line turns downward in the reverse direction as the train backs up onto a parallel track. Let L_{ext} be the extra distance the train must move backward, in addition to the train length L, before it is out of the

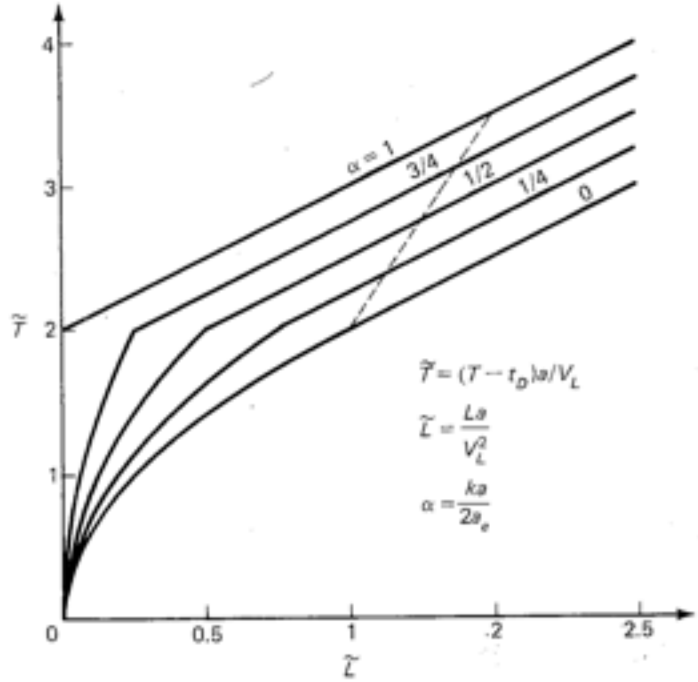


Figure 4-7. The Relationship between Minimum Permissible Headway through Stations and Vehicle Length

way of the next train attempting to enter the station, as shown in figure 4-8.

The minimum L_{mex} can be found from the theory of section 3.4. For a train, the displacement between parallel tracks, H, is generally greater than the value $2a_n$ given by equation (3.4.5). Therefore, as indicated below this equation, a straight section must be inserted between spiral segments. Its length, using equation (3.4.6), is $(H - 2a_n)(4V/2H)$. Therefore the total transition length is $L_{mex} = 4V\cancel{(1.5 - a_n/H)}$. For $a_n = 1.25$ m/s² and, say, $V = 20$m/s and $H = 4$m; $L_{mex} = 95$m.

Consider figure 4-8. Back-up stations are generally considered with train systems in which there are up to ten cars per train. Thus, if each car is say 20 m long, L is on the order of 200 m. The speed V_L corresponding to a stopping distance of $L + L_{mex} = 295$ m is

$$\sqrt{2a(L + L_{mex})} = 27 \text{ m/s} = 61 \text{ mi/h}$$

Thus if V_L is less than this value, which will usually be the case, the train begins to stop inside the distance $L + L_{mex}$. Therefore, the time required to

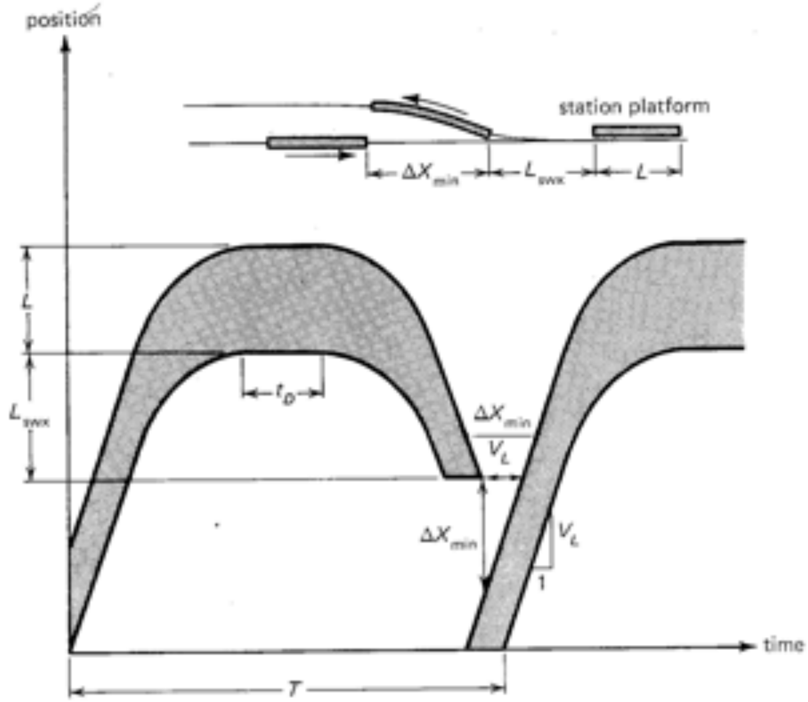


Figure 4-8. The Position-Time Diagram for an End-of-Line Station

stop from a distance $L + L_{swx}$ can be found from equation (4.2.4) by substituting $x_1 = L + L_{swx}$. Then from figure 4-8,

$$T = t_D + 2\left(\frac{L + L_{swx}}{V_L} + \frac{V_L}{2a}\right) + \frac{\Delta x_{min}}{V_L}$$

The velocity of the train approaching the station is still V_L when its front is L_{swx} from the station platform. Therefore substitute Δx_{min} in the above equation from equation (4.2.8) with $V_{min} = V_L$. Then

$$T = t_D + \frac{V_L}{a}(1 + \alpha) + \frac{2}{V_L}(L + L_{swx}) \qquad (4.2.18)$$

in which α is given by equation (4.2.13). Equation (4.2.18) gives the time headway between trains if the trains are travelling at a constant speed V_L when the front passes the point a distance L_{swx} in front of the platform. After this point, the train begins slowing down.

Since V_L is in the numerator of the second term in equation (4.2.18) and in the denominator of the third term, $T(V_L)$ possesses a minimum point. By setting $\partial T/\partial V_L = 0$, the minimum point is seen to correspond to

$$V_L = \sqrt{\frac{2a(L + L_{sux})}{1 + \alpha}} \tag{4.2.19}$$

Substituting equation (4.2.19) into equation (4.2.18),

$$T_{\min} = t_D + 2\sqrt{\frac{2(1 + \alpha)}{a} \, (L + L_{sux})} \tag{4.2.20}$$

Consider a numerical example. If $k = 2$ and $a_e = a$ (typical of train systems), $t_D = 15$ s, and $L + L_{sux} = 230$ m, T reaches the minimum value $T_{\min} = 69.3$ s if $V_L = 17.0$ m/s. By comparison, from equation (4.2.18), if $V_L = 22$ m/s, $T = 71.1$ s, or if $V_L = 12$ m/s, $T = 72.5$ s. Thus a minimum headway of say 75 s is applicable over a wide range of speeds.

In terms of the dimensionless variables given by equations (4.2.13), (4.2.15) and (4.2.16), equation (4.2.18) becomes

$$\bar{T} = 1 + \alpha + 2(\bar{L} + \bar{L}_{sux}) \tag{4.2.21}$$

Comparing with equation (4.2.17), one can see that with back-up stations, the headway increases twice as rapidly with train length at a given speed than with flow-through stations. If V_L is well above the minimum value given by equation (4.2.19), a penalty in capacity for back-up stations can be avoided by reducing V_L as the end station is approached. Thus, in equation (4.2.18), reduce V_L enough so that T computed from this equation equals the value computed from equation (4.2.10) with the normal V_L. This procedure will add to the round-trip time of the vehicles but will maintain capacity, if necessary. In determining the loop or line-haul system minimum headway, if different values of T are found due to different conditions at different stations, clearly the largest headway determines the system capacity. Note that nominal headway is constant around the loop.

4.3 Loop Systems

Consider a loop transit system of arbitrary shape as shown in figure 4-9. Let there be n stations numbered in the direction of flow, and let the distance between the ith and $(i+1)$th stations be $\ell_{i,i+1}$. The stations may be either on line or off line, and the vehicles may run singly or in trains.

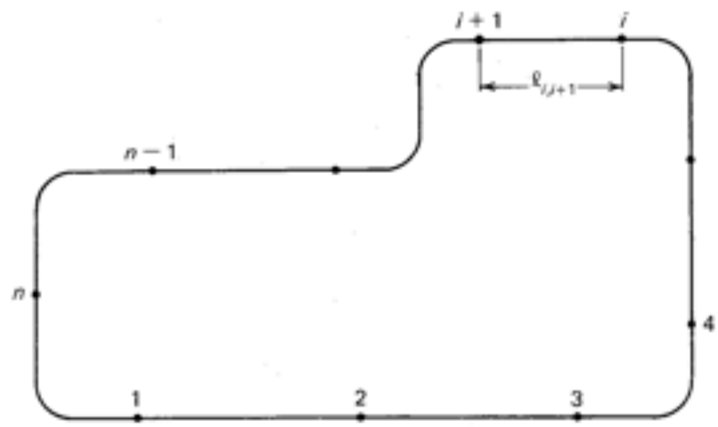


Figure 4-9. Schematic Diagram of a One-Way Transit System

The Maximum Number of Vehicles or Trains

If the stations are on line, the average velocity of a vehicle or train, V_{av}, is given by figure 2-4 as a function of station spacing, station dwell time, line speed, and acceleration level; and the trip time between stations is given by equation (2.4.3). Therefore, the time required to travel completely around the loop is

$$T_Q = nT_{ex} + \frac{\ell_Q}{V_L} \tag{4.3.1}$$

in which T_{ex} is the excess time given by

$$T_{ex} = t_D + \frac{V_L}{a_m} + \frac{a_m}{J_1} \tag{4.3.2}$$

and

$$\ell_Q = \sum_{i=1}^{n} \ell_{i,i+1} \tag{4.3.3}$$

is the distance around the loop. Hence, the average velocity around the loop is

$$V_{av_Q} = \ell_Q/T_Q \tag{4.3.4}$$

With on-line stations, figure 4-6 gives the minimum permissible time headway T_{min} between vehicles or trains. Therefore the maximum number of vehicles or trains that can be accommodated is

$$N_{max} = \frac{\ell_Q}{V_{av_Q} T_{min}} = \frac{T_Q}{T_{min}} \tag{4.3.5}$$

In practical daily operation it is doubtful if the actual number of units will be more than half N_{max}.

If the stations are off line, the average velocity can be found from figure 2-4 if D_s is interpreted as the average trip length. However, the maximum number of vehicles that can be accommodated on line is now based on the on-line speed V_L. The theory of minimum headway is developed in detail in chapter 7, but for most purposes it can be expressed adequately by the equation

$$T_{\min} = \frac{L + H}{V_L} \tag{4.3.6}$$

in which L is the vehicle length and H is the minimum rear-to-front spacing between vehicles, given by

$$H = V_L t_c + \frac{V_L^2}{2}\left(\frac{1}{a_e} - \frac{1}{a_f}\right) \tag{4.3.7}$$

Then $T_{min} = t_c + L/V_L + (V_L/2)(1/a_e - 1/a_f)$

in which t_c is the time constant for application of the braking force that produces the emergency deceleration a_e, and a_f is the failure deceleration rate. Using equation (4.3.6) and V_L for V_{av_Q} in equation (4.3.5), the maximum permissible number of vehicles is found for loop systems with off-line stations. Again, the practical maximum number of vehicles may be less by a factor of two. With off-line stations, the maximum achievable throughput of the stations is given by figure 4-6. If the vehicles operate in platoons, L is the platoon length. Station throughput with off-line stations has been treated by computer simulation by several investigators, the work of which is reported in the book *Personal Rapid Transit II* (see note 3 in chapter 1 of this book).

The Trip-Time and Demand Matrices

For each origin station i in figure 4-9, there are $n - 1$ possible destinations. Therefore it is useful in the following analysis to represent all of the trip times in the form of a matrix.

$$T_{ij} = \begin{pmatrix} T_{11} & T_{12} & T_{13} & \dots & T_{1n} \\ T_{21} & T_{22} & T_{23} & \dots & T_{2n} \\ T_{31} & T_{32} & T_{33} & \dots & T_{3n} \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ T_{n1} & T_{n2} & T_{n3} & \dots & T_{nn} \end{pmatrix} \tag{4.3.8}$$

The first index represents an origin station, and the second a destination. Thus, for example, T_{25} is the time required to travel from station 2 to station 5 counting station dwell time of the vehicles but not the time the patron must wait for a vehicle. It would be trivial to let the major diagonal terms represent the non-trip, therefore let $T_{kk} = T_Q$ for $k = 1, \ldots, n$, where T_Q is the time for a complete circuit given by equation (4.3.1). Note that the diagonal terms of the form $T_{i,i+1}$ and T_{n1} represent the set of n trip times from one station to the next.

The trip time, not counting the time the patron must wait for a vehicle, is given by equation (2.4.3), where D_s is the distance between stops. In on-line station systems

$$T_{ij} = T_{i,i+1} + T_{i+1,i+2} + \ldots + T_{j-1,j}$$

If we let the excess time in equation (2.4.3) due to station dwell and acceleration be as given by equation (4.3.2), then

$$T_{ij} = (j - i)T_{ex} + \frac{\ell_{ij}}{V_L} \qquad \text{(on-line)} \qquad (4.3.9)$$

In systems with off-line stations and nonstop travel from origin to destination

$$T_{ij} = T_{ex} + \frac{\ell_{ij}}{V_L} \qquad \text{(off-line)} \qquad (4.3.10)$$

Some systems have off-line stations but an elevator-type service in which the vehicle can be called into a station on demand and the ride shared. In these systems, T_{ij} is not unique and each case must be treated individually.

Travel demand in person-trips per hour can also be represented by a matrix

$$[D_{ij}]$$

in which the index i represents the trip origin and j the destination. The major diagonal D_{kk} represents the round trip and is zero unless recreational trips are included. There are no simple general relationships among the D_{ij}; however, special cases such as uniform demand in which all the D_{ij} are the same will be treated to gain some insight. Let

$$D_{i\sigma} = \sum_{j=i+1}^{n-1+i} D_{ij} \qquad (4.3.11)$$

and

$$D_{\sigma j} = \sum_{i=j+1}^{n-1+j} D_{ij} \tag{4.3.12}$$

By understanding the meaning of D_{ij}, one can see that $D_{i\sigma}$ is the total flow in people per hour into station i and requesting service on the system. $D_{\sigma j}$ is the total number of people per hour terminating their trips at station j. In general the matrix D_{ij} is a function of time; however, to determine the number of vehicles required it can be assumed independent of time with the terms representing the traffic in the busiest period.

In terms of D_{ij}, the total flow in link i, $i + 1$ in people per hour can be expressed in the form

$$F_{i,i+1} = \sum_{j=i+1}^{n+i-1} D_{ij} + \sum_{j=i+1}^{n+i-2} D_{i-1,j} + \ldots + D_{i+2,i+1}$$

$$= \sum_{k=0}^{n-2} \sum_{j=i+1}^{n+i-k-1} D_{i-k,j} \tag{4.3.13}$$

$c_v = (F_{i,i+1})_{max} T$

The average line flow is

$F_{av} = \frac{\sum_{i=1}^{n} l_{i,i+1} F_{i,i+1}}{l_a}$

$$F_{av} = \frac{1}{n} \sum_{i=1}^{n} F_{i,i+1} \text{ people per hour} \tag{4.3.14}$$

If, in an on-line station system, the headway T is known, the average number of people per vehicle (or per train if vehicles are coupled) is simply

$$p_v = F_{av} T \tag{4.3.15}$$

In an on-line station system, the maximum number of people per vehicle is the maximum $F_{i,i+1}$ multiplied by T.

The Average Trip Length in One-Way Loops

This is a useful concept if it is interpreted as the average weighted in accordance with the amount of travel, that is, let

Average trip length = $\langle L_t \rangle$ = passenger-miles per passenger

Thus

$$= \frac{1}{\bar{D}} \sum_{i=1}^{n} F_{i,i+1} \ell_{i,i+1}$$

$$<L_t> = \frac{\sum_{i=1}^{n} \sum_{j=i+1}^{n-1+i} D_{ij}\ell_{ij}}{\sum_{i=1}^{n} \sum_{j=i+1}^{n-1+i} D_{ij}} \approx \frac{F_{av}\ell_a}{\bar{D}} \qquad (4.3.16)$$

in which n is subtracted from any index $n - 1 + i$ greater than n, and

$$\ell_{ij} = \sum_{k=i}^{j-1} \ell_{k,k+1} \qquad (4.3.17)$$

Consider the case of uniform flow, in which all of the D_{ij} are the same. They can then be factored out of the numerator and denominator of equation (4.3.16). Then if equation (4.3.17) is substituted into equation (4.3.16),

$$<L_t>_{uf} = \frac{1}{n(n-1)} \sum_{i=1}^{n} \sum_{j=i+1}^{n-1+i} \sum_{k=i}^{j-1} \ell_{k,k+1} \qquad (4.3.18)$$

By writing out the terms one can see that

$$\sum_{j=i+1}^{n-1+i} \sum_{k=i}^{j-1} \ell_{k,k+1} = \sum_{k=0}^{n-2} \sum_{j=0}^{k} \ell_{i+j,i+j+1}$$

in which the dummy indices have different meanings on the two sides. The advantage of the new form is that the index i no longer appears in the summation limits. Thus, because of the commutative property of ordinary addition, the outer summation in equation (4.3.18) can be brought inside so that

$$<L_t>_{uf} = \frac{1}{n(n-1)} \sum_{k=0}^{n-2} \sum_{j=0}^{k} \sum_{i=1}^{n} \ell_{i+j,i+j+1}$$

But

$$\frac{1}{n} \sum_{i=1}^{n} \ell_{i+j,i+j+1} = \ell_s \qquad (4.3.19)$$

where ℓ_s is the average distance between stations and it is recognized that the result is independent of the index j. Thus

$$\langle L_t\rangle_{uf} = \frac{\ell_s}{n-1}\sum_{k=0}^{n-2}\sum_{j=0}^{k} 1 = \frac{\ell_s}{n-1}\,(1 + 2 + 3 + \ldots + n - 1)$$

But, by adding the arithmetic series to itself written backwards, the well known result

$$\begin{array}{ccccccccc} & 1 & + & 2 & + & 3 & + \ldots + & (n-1) & \\ + & (n-1) & + & (n-2) & + & . & .. + & 1 & \\ \hline & n & + & n & + & n & + \ldots + & n & = (n-1)n \end{array}$$

is obtained. Thus

$$\langle L_t\rangle_{uf} = \frac{\ell_s n}{2} \qquad (4.3.20)$$

The Average Trip Length in Two-Way Loops

If the loop system of figure 4-9 provides for flows of vehicles in both directions, it can be assumed that each patron will opt to travel the shortest route to his destination. If attention is focused on one of these directions, say the counterclockwise direction shown in figure 4-9, the demand is zero for trips more than half way around the loop. Thus, instead of equation (4.3.16), the average trip length is

$$\langle L_t\rangle = \frac{\displaystyle\sum_{i=1}^{n}\sum_{j=i+1}^{m+i} D_{ij}\ell_{ij}}{\displaystyle\sum_{i=1}^{n}\sum_{j=i+1}^{m+i} D_{ij}} \qquad (4.3.21)$$

in which the limit index m replaces $n-1$. If the stations are approximately equally placed and n is odd, $m = (n-1)/2$. If n is even, the most remote station is just as far away in either direction. Therefore split the demand to it in half. Thus

$$\langle L_t\rangle = \frac{\displaystyle\sum_{i=1}^{n}\left(\sum_{j=i+1}^{i+n/2-1} D_{ij}\ell_{ij} + \tfrac{1}{2}D_{i,i+n/2}\ell_{i,i+n/2}\right)}{\displaystyle\sum_{i=1}^{n}\left(\sum_{j=i+1}^{i+n/2-1} D_{ij} + \tfrac{1}{2}D_{i,i+n/2}\right)} \qquad (4.3.22)$$

With uniform flow and an odd number of stations, by following the process which led to equation (4.3.20) it can be verified that

$$<L_t>_{uf} = \frac{\ell_s(n+1)}{4} \qquad (n \text{ odd}) \qquad (4.3.23)$$

Similarly, equation (4.3.22) becomes

$$<L_t>_{uf} = \frac{\ell_s}{4} \frac{n^2}{(n-1)} \qquad (n \text{ even}) \qquad (4.3.24)$$

The average trip lengths for one-way and two-way systems with uniform flow are summarized in table 4-1.

Table 4-1 Average Trip Lengths

Number of Stations		:	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
$\frac{<L_t>}{l_s}$	One-way	:	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
	Two-way	:	1	1	1.38	1.5	1.8	2	2.29	2.5	2.78	3	3.27
One-way/two-way		:	1	1.5	1.5	1.67	1.67	1.75	1.75	1.8	1.8	1.83	1.83

The Station Delay Time, t_D

Introduce handout

Station delay time is a very important parameter in determining the performance of automated transit systems. It is clearly dependent upon vehicle configuration and flow. If the vehicle has only three seats abreast, simple timing of the exit and entry maneuver shows that five or six seconds may be adequate for t_D. With six seats, three forward and three backward and one door, it may take roughly twice as long to vacate and reload a vehicle. In larger vehicles somewhat less time per person per door is required, and the result depends on the width of the doorway.

The average walk speed is about two miles per hour or three feet per second, therefore the maximum rate of discharge of passengers per door is roughly one per second, one abreast, or two per second, two abreast. These kinds of considerations tempered with simple experiments and observations at transit stations can determine the mean time required for egress and ingress for a given vehicle configuration. Unfortunately, at the time of writing the author cannot point to any literature that presents data on passenger flow in and out of vehicles. Accepted standard values of station dwell are needed for the purpose of predicting the performance of various types of transit systems.

The Required Vehicle Fleet Size

The required size of the vehicle fleet is given by

$$N = N_o + N_e + N_m \tag{4.3.25}$$

in which

- N_o is the required number of occupied vehicles needed to meet the peak demand if there are p_v people per vehicle;
- N_e is the number of empty vehicles in circulation during the peak demand period as a result of nonuniform demand; and
- N_m is the size of the maintenance float, that is, the number of extra vehicles required to account for the possibility of rush period breakdowns.

The number of occupied vehicles, N_o, is simply the number of people riding at any one time during the peak period, divided by the average number of people per vehicle, p_v. The number of people riding at any one time is the peak period flow in people per unit time multiplied by the average trip time. The peak period flow used to determine N_o must be averaged over an accepted period such as fifteen minutes or one hour. If a shorter period is used for averaging, a larger fleet will result, but the average wait time for service in the peak period will be reduced.

It is a policy decision to balance the desire for minimum wait with the added cost in vehicles needed to provide it. For the sake of economy, a certain measure of staggering of demand is needed. Any transit system can be swamped at some time by too great a demand, and the author's experience is that the public understands this and will either accept the need to wait longer in unusually busy periods or individually adjust their schedules to avoid the busiest periods. With these thoughts in mind, the author recommends that in public transit applications, the peak flow for computation of N_o be obtained as the average flow over the busiest hour. On the other hand, if the application is to carry students between classes in which the break period is say 15 minutes, then the peak flow used to compute N_o must be the flow averaged over the time period between the earliest and latest arrivals at the stations that permit the students to arrive at the next class on time. This is an interval of approximately seven minutes if the break period is fifteen minutes. If the starting times between classes are say thirty minutes apart, obtained by staggering class schedules on different campuses, then the average can be taken over a period of 30 − 8 = 22 minutes, instead of 15 − 8 = 7 minutes. Thus, by such a change in class scheduling, the peak flow is reduced to 32 percent of its former value.

Using the notation of the demand and trip-time matrices,

$$N_o = \frac{1}{p_v} \sum_{i=1}^{n} \sum_{j=i+1}^{i+n-1} D_{ij} T_{ij} \quad \text{(one-way)} \qquad (4.3.26)$$

in which the terms of the demand matrix are averaged over an appropriate peak period as discussed above, and one-way flow is assumed. Let

$$\bar{D}_{\text{peak}} = \sum_{i=1}^{n} \sum_{j=i+1}^{i+n-1} D_{ij} \qquad (4.3.27)$$

be the average peak flow on the whole loop system, regardless of flow direction. Then, using equations (4.3.9), (4.3.10), and (4.3.16), equation (4.3.26) can be written

$$N_o = \frac{1}{p_v} \overbrace{\left[\gamma T_{ex} + \frac{<L_t>}{V_L} \right]}^{\text{AV TRIP TIME}, \; T_{AV}} \bar{D}_{\text{peak}} \quad \text{(one-way)} \qquad (4.3.28)$$

in which $<L_t>$ is given in general by equation (4.3.16), and for the case of uniform flow by equation (4.3.20). In on-line station systems,

$$\gamma = \frac{1}{\bar{D}_{\text{peak}}} \sum_{i=1}^{n} \sum_{j=i+1}^{i+n-1} (j - i) D_{ij} \qquad (4.3.29a)$$

and, in off-line station, nonstop systems,

$$\gamma = 1 \qquad (4.3.29b)$$

In the case of uniform flow, D_{ij} is the same for all i and j. Then equation (4.3.27) becomes

$$\bar{D}_{\text{peak}} = n(n - 1) D_{ij} \qquad (4.3.30)$$

and

$$\gamma_{\text{on-line}} = \frac{1}{n(n-1)} \sum_{i=1}^{n} \sum_{j=i+1}^{i+n-1} (j-i)$$

$$= \frac{1}{n(n-1)} \left[\sum_{i=1}^{n} \sum_{j=i+1}^{i+n-1} j - \sum_{i=1}^{n} i \sum_{j=i+1}^{i+n-1} 1 \right]$$

$$= \frac{1}{n(n-1)} \left\{ \sum_{i=1}^{n} \left[i(n-1) + \sum_{j=1}^{n-1} j \right] - (n-1) \sum_{i=1}^{n} i \right\}$$

$$= \frac{1}{n(n-1)} \left(\sum_{i=1}^{n} 1 \right) \left(\sum_{j=1}^{n-1} j \right) = \frac{n}{2} \tag{4.3.31}$$

The ratio of the number of vehicles required in an on-line station system to the number required in an off-line station system is of interest. In the case of uniform flow and one-way loop traffic, this ratio is found from equations (4.3.28), (4.3.29b), and (4.3.31). Thus

$$\frac{N_o(\text{on-line})}{N_o(\text{off-line})} = \frac{\dfrac{n}{2} + \dfrac{<L_t>/V_L}{T_{ex}}}{1 + \dfrac{<L_t>/V_L}{T_{ex}}} \tag{4.3.32}$$

Consider a typical example in which $V_L = 15$ m/s, $a_m = J_1 = 2.5$ m/s, and $t_D = 15$ s. Then, from equation (4.3.2), $T_{ex} = 22$ s. Let $n = 7$ stations and $<L_t> = 1.5$ mi $= 2400$ m. Then $<L_t>/V_L T_{ex} = 7.27$, and the ratio of equation (4.3.32) is 1.30. Thus, in this case, if all parameters are equal, an on-line station system requires 30 percent more occupied vehicles to serve a given demand than an off-line station system.

In two-way systems, the number of vehicles on each track is obtained from an equation analogous to equation (4.3.26) if the upper limit on the inner sum is changed as in equations (4.3.21) and (4.3.22). If the demand is roughly equal in the two directions, and n is odd, the total number of occupied vehicles required in both directions is

$$N_o = \frac{2}{p_v} \left[\gamma T_{ex} + \frac{<L_t>}{V_L} \right] \sum_{i=1}^{n} \sum_{j=i+1}^{i+(n-1)/2} D_{ij}$$

$$= \frac{1}{p_v} \left[\gamma T_{ex} + \frac{<L_t>}{V_L} \right] \bar{D}_{\text{peak}} \text{ (two-way)} \tag{4.3.33}$$

in which $<L_t>$ is given by equation (4.3.21) in general, or by equation (4.3.23) in the case of uniform flow. With off-line stations, $\gamma = 1$ as before, and with on-line stations and n odd,

$$\gamma = \frac{\sum_{i=1}^{n} \sum_{j=i+1}^{i+(n-1)/2} (j-i) D_{ij}}{\sum_{i=1}^{n} \sum_{j=i+1}^{i+(n-1)/2} D_{ij}} \quad \begin{array}{c}(n \text{ odd}) \\ \text{two-way}\end{array} \qquad (4.3.34)$$

In the uniform flow case, by following the derivation of equation (4.3.31), it can be verified that, if n is odd,

$$\gamma = \frac{n+1}{4} \qquad (n \text{ odd}) \qquad (4.3.35)$$

The computation of N_0 is summarized in table 4-2 (T_{ex} is always found from equation (4.3.2)), and is based on the form of equation (4.3.28).

Returning to equation (4.3.25), consider the computation of N_e. In on-line station systems, service is scheduled and the concept of dispatching

Table 4-2 Computation of the Required Number of Occupied Vehicles

Case	*Flow Directions*	*Stations*		γ	$<L_t>$	$\dot{D}_{peak}$
General	One-way	On-Line		(26*a*)	(4.3.16)[a]	(4.3.27)
		Off-Line		1	(4.3.16)	(4.3.27)
	Two-Way	On-Line	odd no.	(34)	(4.3.21)	(4.3.27)
			even no.	(19)[b]	(4.3.22)	(4.3.27)
		Off-Line	odd no.	1	(4.3.21)	(4.3.27)
			even no.	1	(4.3.22)	(4.3.27)
Uniform Flow	One-Way	On-Line		$n/2$	(4.3.20)	(4.3.30)
		Off-Line		1	(4.3.20)	(4.3.30)
	Two-Way	On-Line	odd no.	$(n+1)/4$	(4.3.23)	(4.3.30)
			even no.	$\frac{n^2}{4(n-1)}$	(4.3.24)	(4.3.30)
		Off-Line	odd no.	1	(4.3.23)	(4.3.30)
			even no.	1	(4.3.24)	(4.3.30)

[a]Numbers in parentheses are equation numbers.

[b]Obtained from equation 4.3.22 by the substitutions l_{ij} becomes $j - i$; $l_{1,1+n/2}$ becomes $n/2$.

empty vehicles to meet demands is not applicable, therefore all vehicles are considered occupied, and $N_e = 0$. The number of empty vehicles required in off-line station systems is zero if the demand is completely uniform, and depends on the nonuniformity in demand. Let EX_{vj} be the excess flow of vehicles into station j, that is, the number of vehicles in excess of those needed to meet the demand for service at station j. Then, in one-way systems,

$$EX_{vj} = \frac{1}{p_v} \sum_{i=j+1}^{n+j-1} (D_{ij} - D_{ji}) \sim \frac{D_{Tj} - D_{jT}}{p_v} \quad (4.3.36)$$

If $EX_{vj} > 0$, there is an excess of vehicles at station j; and if $EX_{vj} < 0$ there is a shortage. If the quantities EX_{vj} are known for all j, they can then be used to compute a schedule of empty vehicle dispatching commands to provide vehicles where needed. The total excess of vehicles is

$$\sum_{j=1}^{n} EX_{vj} = 0 \quad (4.3.37)$$

as may be seen by noting from equation (4.3.27) that $\bar{D}_{\text{peak}}$ can be found by first summing over all destinations from a given origin, or first over all origins to a given destination. Therefore, the problem is one of optimal redistribution of empties.

The number of empty vehicles required is determined by summing the products

$$EX_{vj} T_{ji}$$

in which j corresponds to the stations for which $EX_{vj} > 0$, and T_{ji} is the time required for these vehicles to reach their destination stations. This is a logic operation and can be written in general in the form of a computer program but not neatly in an equation. It is more transparent, however, to consider each case directly once the EX_{vj} have been computed from the demand matrix. Consider the counterclockwise loop system illustrated in figure 4-10.

The small integers are the station numbers and the bold number next to station i is EX_{v_i} in vehicles per minute. In a loop system with off-line stations, under the restriction that no empty vehicle travels the full circuit,

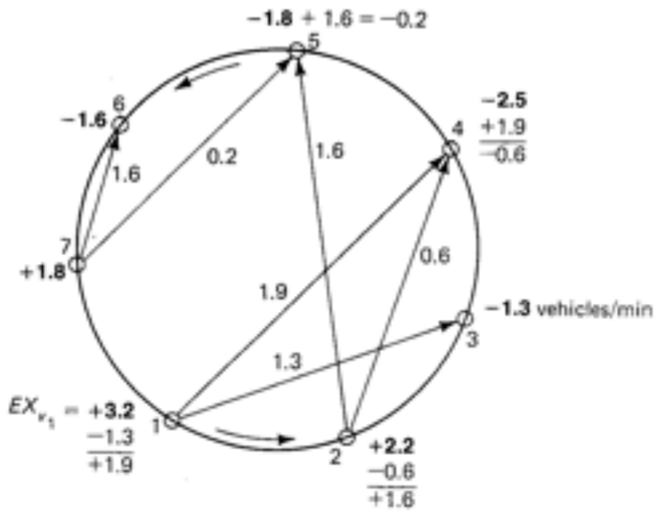


Figure 4-10. Example Computation of the Empty-Vehicle Fleet

it makes no difference in the total number of empty vehicle miles travelled to which stations the excess vehicles are dispatched. Then, consider the dispatching schedule. Arbitrarily start with station 1. If $EX_{v_1} > 0$, as is true in the example of figure 4-9, mentally start 3.2 vehicles per minute (one vehicle every 18.7 seconds) moving around the loop looking for vehicle shortages. The first "sink" ($EX_{v_i} < 0$) is station 3 which demands 1.3 vehicles per minute. Since $EX_{v_1} > |EX_{v_3}|$, station 1 can supply all vehicles needed at station 3. At station 1, subtract 1.3 v/min from the total excess to get 1.9 v/min remaining. Station 4 can use more than this number, therefore dispatch 1.9 v/min from station 1 to station 4. Deducting this number from EX_{v_4} = -2.5 leaves -0.6 v./min. All vehicles from station 1 have found destinations. Therefore move to station 2 and repeat the process. Then move to station 3 and note that its requirement is satisfied. Similarly, the need for vehicles at station 4 is satisfied. The remaining shortages at station 5 and 6 are then made up by circulation of vehicles from station 7. By drawing flow lines from "sources" to "sinks" and labelling them with the vehicle flows, an equation for N_e can be written directly from the diagram. Thus

$$N_e = 1.3T_{13} + 1.9T_{14} + 0.6T_{24} + 1.6T_{25} + 0.2T_{75} + 1.6T_{76}$$

Using equation (4.3.10), $T_{ij} = T_{ex} + \ell_{ij}/V_L$,

$$N_e = T_{ex}\sum_{+} EX_{vj} + 1/V_L \sum \mathrm{FLOW}_{ij}\ell_{ij} \tag{4.3.38}$$

in which $+EX_{vj}$ is the sum of all positive EX_{vj}, and in the example is 7.2 v/m. The second sum, in the example, is

$$
\begin{aligned}
\sum \mathrm{FLOW}_{ij}\ell_{ij} = \;& 1.3(\ell_{12} + \ell_{23}) \\
& +1.9(\ell_{12} + \ell_{23} + \ell_{34}) \\
& +0.6(\qquad \ell_{23} + \ell_{34}) \\
& +1.6(\qquad \ell_{23} + \ell_{34} + \ell_{45}) \\
& +0.2(\ell_{12} + \ell_{23} + \ell_{34} + \ell_{45} + \qquad \ell_{71}) \\
& +1.6(\ell_{12} + \ell_{23} + \ell_{34} + \ell_{45} + \ell_{56} + \ell_{71}) \\
= \;& 5.0\ell_{12} + 7.2\ell_{23} + 5.9\ell_{34} + 3.4\ell_{45} + 1.6\ell_{56} + 1.8\ell_{71}
\end{aligned}
$$

If the flow is two-way, equation (4.3.36) is replaced by a pair of equations, one for counterclockwise flow in the directions of the indices shown in figure 4-9, and the other for clockwise flow. Thus,

$$
\left.
\begin{aligned}
\overset{\curvearrowleft}{EX}_{vj} &= \frac{1}{P_v}\left(\sum_{i=j-m}^{j-1} D_{ij} + \tfrac{1}{2}D_{j-n/2,j} - \sum_{i=j+1}^{j+m} D_{ji} - \tfrac{1}{2}D_{j,j-n/2} \right) \\
\overset{\curvearrowright}{EX}_{vj} &= \frac{1}{P_v}\left(\sum_{i=j+1}^{j+m} D_{ij} + \tfrac{1}{2}D_{j-n/2,j} - \sum_{i=j-m}^{j-1} D_{ji} - \tfrac{1}{2}D_{j,j-n/2} \right)
\end{aligned}
\right\}
\tag{4.3.39}
$$

in which

$$
\begin{aligned}
m &= \frac{n-1}{2} \qquad && \text{for } n \text{ odd} \\
&= \frac{n}{2} - 1 \qquad && \text{for } n \text{ even}
\end{aligned}
$$

and the terms not under a summation sign are dropped if n is odd. If an index is greater than n, n is subtracted from it; and if an index is less than 1, n is added to it. Based upon equations (4.3.39), the procedure for determining the size of the empty fleet is the same as in the case of one-way loops.

Returning again to equation (4.3.25), consider the computation of N_m. Scheduled maintenance should be done in the off-peak hours, and then does not enter into N_m. The fleet N_m is needed rather to maintain $N_o + N_e$ vehicles in operation during the peak period even though some vehicles may fail and require unscheduled maintenance. Assume that if a vehicle fails during a rush period, it can be returned to service in a time *MTTR*, that is, the mean time to repair. *MTTR* is made up of the following components:

$$\begin{aligned} MTTR = &\ (\text{mean time to dispatch vehicle to maintenance}) \\ &+ (\text{mean time to ready vehicle for repair} \\ &\quad \text{including time to obtain needed parts}) \\ &+ (\text{mean time to replace faulty part or subsystem}) \\ &+ (\text{mean time to dispatch vehicle back in service}) \end{aligned}$$

Let the mean time between vehicle failures be *MTBF*, and let T_{rush} be the length of the rush period. Then the number of vehicles that fail during the rush period is

$$(N_o + N_e)\,(T_{rush}/MTBF)$$

If *MTTR* is of the order of T_{rush} but not so long that the vehicle cannot be restored to service by the next rush period,

$$N_m = (N_o + N_e)\,(T_{rush}/MTBF) \tag{4.3.40a}$$

But, if *MTTR* is much less than T_{rush},

$$N_m = (N_o + N_e)\,(MTTR/MTBF) \tag{4.3.40b}$$

Thus, the importance of easy-maintenance design so that subsystems can be quickly replaced is apparent. Life cycle cost is minimized if an expensive vehicle is returned to service as rapidly as possible. In a well-designed system, N_m should be no more than about one percent of $N_o + N_e$.

The Average Number of People per Vehicle and Time Headway

If p_v is given, $N_o + N_e$ can be determined from the theory of the previous section. Then the average time headway between vehicles, T, is found from equation (4.3.5). Thus

$$T = \frac{\ell_Q}{V_{av_Q}(N_o + N_e)} = \frac{T_Q}{N_o + N_e} \tag{4.3.41}$$

In small-vehicle automated systems in which private party service is offered, p_v is the size of the average group traveling together, and is usually assumed to be about 1.5. In larger vehicle systems, however, the service must be scheduled to a given value of T. Then equation (4.3.41) is used to compute $N_o + N_e$, and equation (4.3.15) is used to compute p_v.

Capacity

The capacity of a loop system is the total number of people per hour the system can handle. The achievable capacity depends on the distribution of demand as characterized by the demand matrix. In on-line station systems, it is limited by the achievable station throughput, derived in section 4.2. In off-line station systems, capacity may be limited by either station throughput or line throughput. If there is only a small number of stations, station throughput, determined by the theory of section 4.2, is the limiting factor. But with a small number of stations, off-line stations often cannot be justified. With a large number of off-line stations, the line capacity, determined by equations (4.3.6) and (4.3.7), limits the system capacity. The line flow in each link can readily be determined from the demand matrix D_{ij}.

4.4 Line-Haul Systems

A line-haul system is a collapsed one-way loop which may have either continuous flow or reverse flow at the end stations. As indicated in section 4.2, loop end stations cut the achievable headway at a given line speed almost in half and hence without a speed change almost double the capacity. But they take more space and are more expensive than back-up end stations. Therefore, the back-up end station is often used. Headway with these can be maintained if the trains are caused to slow down well in advance of the end stations. The intermediate stations may use either side platforms or central platforms, the latter of which are more economical of space. As indicated in figure 4-11, the stations of line-haul systems are usually on line. Also, to obtain adequate capacity, the vehicles are usually trained.

The maximum number of trains is given by equation (4.3.5), in which the minimum headway is found from section 4.2. As with on-line station loop systems, the actual number of trains required is found from equation (4.3.5) with the desired scheduled headway T_h substituted for $T_{\min}$. In on-line station systems, there is no deliberate circulation of empty vehicles; therefore, $N_e = 0$ and N_o is the result found by using equation (4.3.5). Equation (4.3.15) is used to find p_v, the average rush period number of

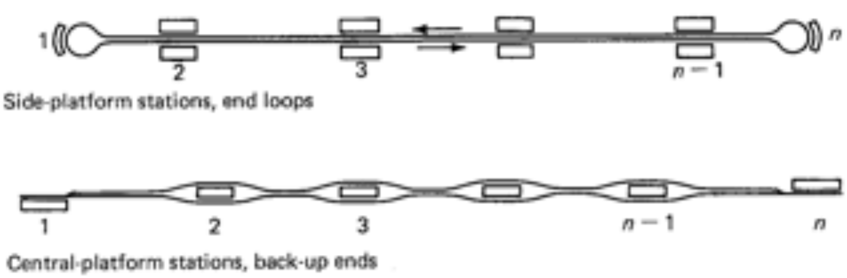


Figure 4-11. Line-Haul Configurations

people per train. In line-haul systems, however, the summation limits are different. Using the station designations of figure 4-11 for flow to the right,

$$p_v = \frac{1}{N_o} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} D_{ij} T_{ij}$$

Substituting equation (4.3.9),

$$p_v = \frac{1}{N_0} \left(\gamma T_{ex} + \frac{<L_t>}{V_L} \right) \hat{D}_{peak} \tag{4.4.1}$$

in which

$$\hat{D}_{peak} = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} D_{ij} \tag{4.4.2}$$

$$\gamma = \frac{1}{\hat{D}_{peak}} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} D_{ij}(j - i) \tag{4.4.3}$$

and

$$<L_t> = \frac{1}{\hat{D}_{peak}} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} D_{ij} \ell_{ij} \tag{4.4.4}$$

In a given peak period, the terms of the demand matrix D_{ij} corresponding to flow to the right in figure 4-11 are usually quite different from those corresponding to flow to the left. Thus p_e will be different in the two cases, and for the purpose of computing the number of cars per train p_e must of course be taken as the larger of the two values. If the car capacity is given, the number of cars per train is

$$\text{No. cars per train} = \frac{p_e}{(\text{car capacity})(\text{load factor})} \qquad (4.4.5)$$

in which "load factor" is the desired fraction of car capacity used during the rush period averaged over all cars in the system headed in the direction of maximum flow.

4.5 Network Systems

A network transit system is one in which there is more than one path between some of the stations. Fixed route, fixed schedule bus systems are usually network systems; the New York subway system is a network system. If network systems involve transfers from one branch to the other, however, they can be considered as being composed of a series of loops or line-haul branches. In these cases, the theory of sections 4.3 and 4.4 can be applied directly, and further elaboration is unnecessary. Thus, the present analysis is restricted to networks in which the vehicles may transfer from one loop or branch to another. Except in very small networks, the economics favor the use of off-line stations because: (1) they allow use of smaller vehicles and lower maximum line speeds, and hence guideways of lower weight per unit length; (2) they permit lower average trip time and hence reduce both the number of vehicles of a given size required and the total vehicle fleet cost; and (3) they increase patronage because of the reduced trip time. Therefore, the network analysis of this section assumes off-line stations.

Networks may use multilevel interchanges to accomplish vehicle transfer or they may use Y-interchanges. Use of Y-interchanges has the advantage that the guideways can all be at one level and the visual presence at any one location is minimized, but the disadvantage that, at interchanges, vehicles must merge before they diverge thus reducing the capacity. With multilevel interchanges, the vehicle streams diverge before they merge, thus preventing bottlenecks. The disadvantage of the Y-interchange can be reduced by designing the system so that vehicles run both above and below the guideway, thus providing two-way traffic. (A side-by-side two-way system is not practical in network configurations because of the size and complexity of the interchanges.) A two-way, over/under system more than doubles capacity with a given set of line and station locations because, as shown in Table 4-1, the average trip length reduces. With reduced trip length, trip time reduces and with it the number of vehicles. Hence the minimum spacing between vehicles increases. The analysis will, however, treat both multilevel and Y-interchanges; and both one-way and two-way networks.

If a specific network is under consideration; that is, a network with specific line and station locations, and the analyst has the data needed to make a detailed performance, cost, and patronage analysis, then the analysis of performance characteristics should proceed by extending the theory of section 4.3 for loop systems. The same basic framework of analysis is still applicable, but the analyst must take into account that the travel time matrix T_{ij} is not unique, but depends upon the choice of route. The network should be designed, however, so that the nominal path minimizes T_{ij}. Nonminimum paths would be used only in abnormal circumstances such as unusually heavy demand on certain routes, or in the case of breakdowns. Equation (4.3.10) is still valid, therefore the minimum T_{ij} are found by first finding the minimum ℓ_{ij}.

The formula for average trip length is analogous to equation (4.3.16) with the summations extending over all stations; and the theory of the required fleet size follows directly. Equation (4.3.26), with the summations again extended over all stations, shows that the minimum T_{ij} produce the minimum fleet size. The computation of the required empty vehicle fleet size proceeds by first computing the EX_{eij} from equation (4.3.36) with new summation limits. Having the EX_{eij}, the choice of destinations to which the vehicles are routed is more complex than in the case of loops. This problem has been treated by Thangavelu[2], by Irving[3], and by others. A trial selection of the empty vehicle destinations can be made on the basis of minimizing the total empty vehicle travel time. Then the total flow on each link must be computed, and the empty vehicle destinations and routes adjusted until a given link capacity constraint is satisfied.

Rough computations of vehicle fleet size can be made on the following basis: If the flow is completely uniform, $D_{ij} = D_{ji}$, and no empty vehicles are required. On the other hand, if the demand is unidirectional in the sense that if $D_{ij} \neq 0$, $D_{ji} = 0$, the occupied vehicles going from i to j must circulate empty from j to i. In this case half of the vehicles are empty. Therefore, the assumption that one-third of the vehicles circulate empty is a good compromise between these extremes. A number of computer simulations have produced the result of approximately one-third empty vehicles.

In initial analysis of network systems, before specific line and station locations are chosen, it is necessary to be able to estimate the performance and economics in specific situations. Also, a theory of performance and economics of networks at this level is algebraically simple, and it is easy to determine the influence of parameters such as line and station spacing. The theory is developed in two parts: geometric parameters, and performance parameters.

Geometric Parameters

The parameters derived are line density (length of guideways per unit area), station density, intersection point density, and average trip length. Consider the idealized network of figure 4-12. Let the network be square with line spacing L and $n + 1$ lines in each direction. Then the total length of lines is

$$\begin{aligned}\mathscr{L} &= 2nL(n + 1)\\ &= \frac{2}{L}(nL)^2\left(1 + \frac{1}{n}\right)\end{aligned}$$

But $(nL)^2 = A$ = the area of the network. Therefore,

$$\mathscr{L} = \frac{2}{L}\beta A \tag{4.5.1}$$

where

$$\beta = 1 + L/A^{1/2} \tag{4.5.2}$$

The line density is therefore

$$\rho_\ell = \frac{\mathscr{L}}{A} = \frac{2}{L}\beta \tag{4.5.3}$$

Let the stations be placed at the midpoints as indicated by the dots in figure 4-12. There are three reasons for this: (1) it is awkward to incorporate stations in the intersections and such a procedure increases visual impact at the intersections; (2) for a given L the maximum rectangular walk distance is $L/2$ if the stations are at the midpoints but twice as great if the stations are at the intersections (Even if the street pattern is not a rectangular grid, the rectangular walk distance is more realistic than the shortest distance "as the crow flies."); and (3) for a given line density, the station density is maximized if the stations are at the midpoints. The third reason will be clear from the following analysis.

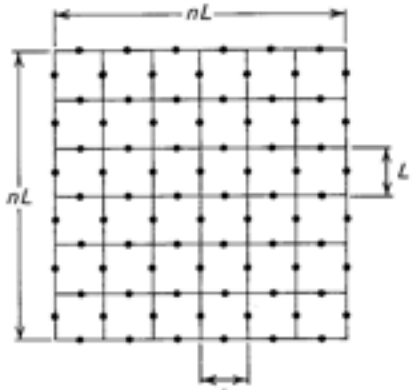


Figure 4-12. An Idealized Transit Network

The number of stations in the network of figure 4-12 is

$$n_s = 2n(n+1) = \frac{2}{L^2}\beta A \tag{4.5.4}$$

The station density is therefore

$$\rho_s = \frac{2}{L^2}\beta \tag{4.5.5}$$

The number of intersections in the network of figure 4-12 is

$$n_I = (n+1)^2 = \frac{1}{L^2}\beta^2 A \tag{4.5.6}$$

Hence the intersection density is

$$\rho_I = \frac{\beta^2}{L^2} \tag{4.5.7}$$

Thus the ratio of station density to intersection density is

$$\rho_s/\rho_I = \frac{2}{\beta} \tag{4.5.8}$$

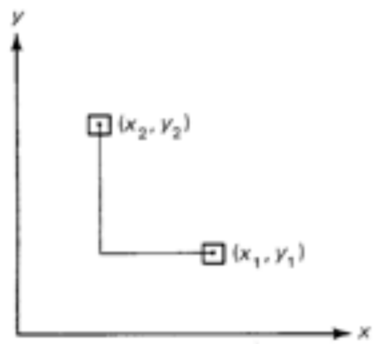


Figure 4-13. Idealization of the Average Trip Length

Thus, for a given line density, the station density is almost twice as great if the stations are at the midpoints rather than at the intersections. On the other hand, for a given station density (the parameter that determines the patronage), the line density is greater by $2/\beta$ if the stations are at the intersections rather than at the midpoints.

Consider the average trip length $<L_t>$ on the network of figure 4-12. If n is large, $<L_t>$ can be approximated by integrating the rectangular trip length over the network area, as shown in Figure 4-13, and by assuming every station is both an equally likely origin for travel and an equally likely destination. This is the assumption of uniform travel as introduced in section 4.3. If n is large, reference to figure 4-13 gives

$$<L_t> = \frac{1}{(nL)^4}\int_0^{nL}\int_0^{nL}\int_0^{nL}\int_0^{nL}(|x_1 - x_2| + |y_1 - y_2|)dx_1\, dy_1\, dx_2\, dy_2$$

$$= \frac{2}{(nL)^2}\int_0^{nL} dx_2 \left[\int_0^{x_2}(x_2 - x_1)dx_1 + \int_{x_2}^{nL}(x_1 - x_2)dx_1\right]$$

$$= \frac{2}{(nL)^2}\int_0^{nL}\left[\frac{(nL)^2}{2} - nLx_2 + x_2^2\right]dx_2$$

$$= \frac{2}{3}(nL) = \frac{2}{3}A^{1/2} \qquad (4.5.9)$$

Thus, in the limit as n approaches infinity, the average trip length with

Figure 4-14. Four-Station Square Loop

uniform demand is two-thirds the square root of the network area. In finite networks, $<L_t>$ is larger than this limit value because of indirect routing. Consider the series of cases illustrated by figures 4-14 through 4-17 with uniform demand.

Figure 4-14 shows the simplest case, consisting of the basic four-station square loop. Let the distance between stations be L. Then for one-way travel,

$$<L_t> = \frac{(1 + 2 + 3)}{3} \quad L = 2L = 2A^{1/2} \qquad (4.5.10)$$

and for two-way travel

$$<L_t> = \frac{(1 + 2 + 1)}{3} \quad L = 4/3 \ L = 1.33A^{1/2} \qquad (4.5.11)$$

$A = 3L^2$

$L = \frac{A}{3}$

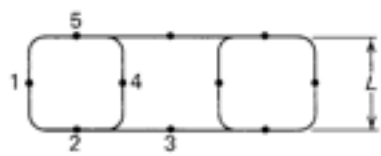


Figure 4-15. A Two-Loop Network

Figure 4-15 shows the next level of complexity. If the flow is *one way,* say counterclockwise, the average trip lengths from each of the five numbered stations are different and are as follows:

Origin Station	$<L_t>_i/L$
1	32/9
2	26/9
3	40/9
4	44/9
5	38/9

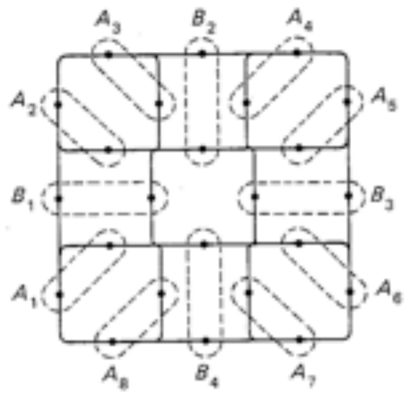


Figure 4-16. A Five-Loop Network

By symmetry, the average trip length from the other five stations are the same. Therefore, the average trip length for the two-loop network with one-way flow and uniform demand is

$$<L_t> = \frac{180}{5(9)} L = 4L = 2.31A^{1/2} \qquad \text{(one-way flow)} \qquad (4.5.12)$$

Similarly

$$<L_t> = \frac{256}{90} L = 2.88L = 1.66A^{1/2} \qquad \text{(two-way flow)} \qquad (4.5.13)$$

The next level of complexity is illustrated in figure 4-16. In this case the process of calculating the trip lengths is complex enough that a systematic procedure is desirable. Let the stations be divided into two types of groups of two stations each: A groups and B groups as shown in figure 4-16. Let $A_i \rightarrow A_j$ represents the trips from each of the stations in group A_i to each of the stations in group A_j, and note that there are four such trips. Then

$$A \rightarrow A = \sum_{i} \sum_{j} A_i \rightarrow A_j \quad i \neq j$$

represents all of the trips between A groups except for the trips internal to each A group. These are denoted by

$$A_I = \sum_{i} A_i \rightarrow A_i$$

The totality of trips in the network of figure 4-16 can be represented by the expression

$$\begin{array}{llllll} A_I + & (A \rightarrow A) + & (A \rightarrow B) + & B_I + & (B \rightarrow A) + & (B \rightarrow B) \\ 16 + & 8(28) + & 8(16) + & 8\ + & 4(32) + & 4(12) = 552 \text{ trips} \end{array}$$

The numbers under the group symbols are the numbers of trips generated in each type of group combination. Since the total number of stations is 24, the total number of trips is 24(23) = 552. The total length of trips in each of the six groups is given in table 4-3 for one-way and two-way flow,

Table 4-3 Computation of Average Trip Length in Five-Loop Network

Group	*Number of Trips*	*Total Length of Trips/L* One-Way Flow	Two-Way Flow
A_I	16	8(4)	8(2)
$A \rightarrow A$	224	8(120)	8(108)
$A \rightarrow B$	128	8(64)	8(54)
B_I	8	4(8)	4(8)
$B \rightarrow A$	128	4(152)	4(112)
$B \rightarrow B$	48	4(60)	4(52)
	552	2384	2000
$<L_t>$		$1.44A^{1/2}$	$1.21A^{1/2}$

in which $A^{1/2} = 3L$.

The network of figure 4-17 has 60 stations and 60(59) = 3540 trips. Using the same types of groupings as in figure 4-16, the average trip lengths are determined in a similar manner and are as given in table 4-4. Recognition of symmetries greatly simplifies the process of counting the trip lengths.

In Figure 4-18, the average trip lengths corresponding to the square networks of figures 4-14, 4-16 and 4-17 are plotted. Also, the limit given by equation (4.5.9) is shown as a dashed line. This information is as much as is useful to obtain for the case of uniform flow.

Performance Parameters

The parameters derived are fleet size, average headway, line flow, station flow, and nonstop wait time. The fleet size is given by equation (4.3.25) and

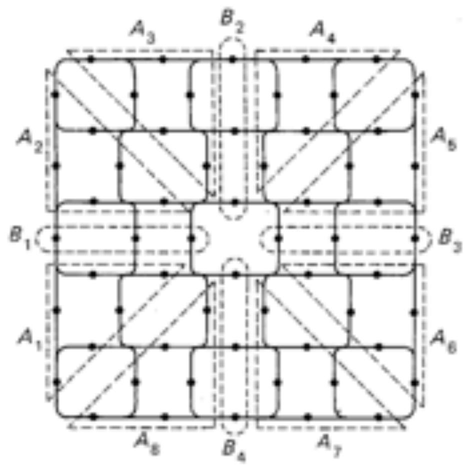


Figure 4-17. A Thirteen-Loop Network

the derivation of each term in that equation proceeds in the same manner as for loops. With networks, however, it is useful to define the parameters

$$\sigma = \frac{N}{N_o + N_e} \tag{4.5.14}$$

$$f_p = \frac{N_o}{N_o + N_e} \tag{4.5.15}$$

Then

$$N = \frac{\sigma N_o}{f_p} \tag{a}$$

Table 4-4 Computation of Average Trip Length in Thirteen-Loop Network

Group	*Number of Trips*	*Total Length of Trips* *One-Way Flow*	*Two-Way Flow*
A_I	8(30)	8(132)	8(98)
$A \rightarrow A$	8(252)	8(3262)	8(1370)
$A \rightarrow B$	8(72)	8(840)	8(326)
B_I	4(6)	4(24)	4(24)
$B \rightarrow A$	4(144)	4(804)	4(648)
$B \rightarrow B$	4(27)	4(142)	4(124)
	3540	21,344	17,536
$<L_t>$		$1.21A^{1/2}$	$0.99A^{1/2}$

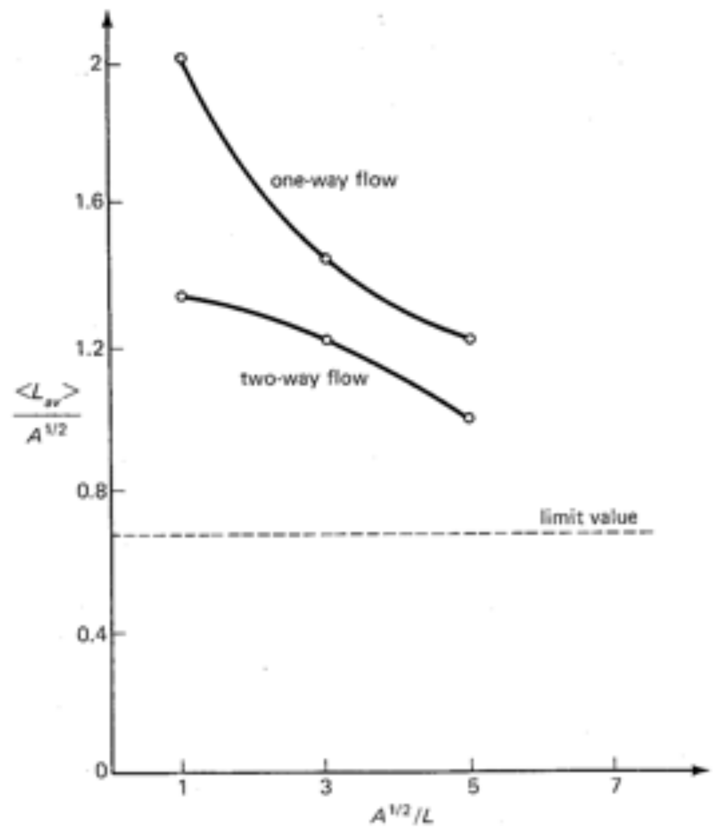


Figure 4-18. The Average Trip Length in Finite Networks with Uniform Demand

In analogy with equation (4.3.26)

$$N_o = (\bar{t}_h A) T_{\mathrm{trip}} / p_v \tag{b}$$

in which $\bar{t}_h$ is the trip density, that is, the number of trips carried on the network system per hour per unit area, A is the network area, and p_v is the average number of people per occupied vehicle. In analogy with equation (4.3.10), the average trip time is

$$T_{\mathrm{trip}} = T_{\mathrm{ex}} + \frac{<L_t>}{V_L} \tag{4.5.16}$$

Combining equations (a), (b), and (4.5.16), the fleet size is

$$N = \frac{\sigma \bar{i}_h}{p_v f_p} \left(T_{ex} + \frac{<L_t>}{V_L} \right) A \qquad (4.5.17)$$

If the vehicle makes intermediate stops, T_{ex} must be multiplied by the average number of intermediate stops, as indicated by equation (4.3.9).

The average time headway, T_{av}, is found from an equation analogous to equation (4.3.5), that is, by observing that the number of vehicles on line, N/σ, is equal to the total line length $\mathscr{L}$ (equation 4.5.1)) divided by the average nose-to-nose spacing between vehicles. The latter quantity is $T_{av}V_{av}$, where V_{av} is the average velocity. Thus, using equations (4.5.1), (4.5.17) and (4.5.16),

$$T_{av} = \frac{\mathscr{L}}{(N/\sigma)V_{av}} = \frac{2\beta p_v f_p}{\bar{i}_h L T_{trip} V_{av}}$$

But $T_{trip}V_{av} = <L_t>$. (Note by combining equations (2.4.5) and (4.3.2) we see that $V_L T_L T_{ex} = \hat{D}$.) Thus

$$T_{av} = \frac{2\beta p_v f_p}{\bar{i}_h L <L_t>} \qquad (4.5.18)$$

and, as can be expected, for a given trip density, T_{av} is independent of A.

The average line flow in people per unit time, f_{av}, is the line flow in vehicles per unit time, $1/T_{av}$, multiplied by the number of people per vehicle, $p_v f_p$. Thus, using equation (4.5.18),

$$f_{av} = \frac{p_v f_p}{T_{av}} = \frac{\bar{i}_h L <L_t>}{2\beta} \qquad (4.5.19)$$

The average station flow in people per unit time, fs_{av}, is the total demand per unit time, $\bar{i}_h A$, divided by the number of stations. Using equation (4.5.4),

$$fs_{av} = \frac{\bar{i}_h A}{n_s} = \frac{\bar{i}_h L^2}{2\beta} \qquad (4.5.20)$$

and it is interesting to observe that

$$f_{av} = fs_{av} \left(\langle L_t \rangle / L\right) \tag{4.5.21}$$

Finally, the nonstop wait time T_{nswt} is the average time a vehicle must wait at a station after one party has boarded for a second party headed for the same destination to arrive and board. This quantity is the average time headway between arrivals of parties at a station, p_v/fs_{av}, multiplied by the number of possible destinations $(n_s - 1)$. Thus

$$T_{nswt} = \frac{p_v(n_s - 1)}{fs_{av}} = \frac{p_v(n_s - 1)n_s}{\bar{i}_h A}$$

For $n_s >> 1$, and using equation (4.5.4),

$$T_{nswt} = \frac{4p_v\beta^2}{\bar{i}_h L^4} A = \frac{4p_v}{\bar{i}_h L^2}\left(\frac{A^{1/2}}{L} + 1\right)^2 \tag{4.5.22}$$

This equation is meaningful if the demand is relatively uniform, but T_{nswt} will in general differ a great deal between station pairs. Note that, for a given trip density, the nonstop wait time increases with the area of the network. Thus, for a given trip density, the type of service that requires a party to wait in a vehicle until a second party boards becomes increasingly unattractive as the network grows. Also, such a service concept increases the total trip time and hence the vehicle fleet size. Other service concepts can be considered. For example, if intermediate stops are permitted and the average number of stops counting the trip end stop is s, the average time to wait at a station for a second party going to one of these s stops is T_{nswt}/s. In another case, if it is desired to increase the vehicle load factor further by waiting for n_p extra parties going to any one of s stops, the wait time of the vehicle and the first party is $n_p T_{nswt}/s$. Thus knowledge of T_{nswt} determines the vehicle wait time for a range of service concepts.

4.6 Summary

The purpose of this chapter has been to develop the theory of performance of various types of transit systems. By "performance" we mean quantities such as characteristic times, trip lengths, average speeds, line flows, station flows, required numbers of vehicles, and average vehicle occupancy. Table 4-5 gives a classification of types of transit systems. Four basic types, classified according to the geometry of the lines—shuttle, loop, line-haul, and network—form the headings of the major sections of the chapter. Each of the four basic types may be further classified according to the geometry of the lines and stations, as indicated in table 4-5, and according to the type of service provided. Dropping the nonapplicable classifications, twenty-five different possibilities remain. In exploring the basic types of systems more deeply, we find that further subclassifications are practical, and discuss these in individual sections.

In section 4.1, shuttle systems are considered. First the simple shuttle is analyzed and it is found that all of its characteristic times can be described in one chart—figure 4-2. Here, based on the distance between the two stations and the line speed, the wait time to call a vehicle from the other end is found. Then, given the average station delay or dwell time, the average wait time, effective headway, and capacity in vehicles per hour are found. Next, the shuttle with intermediate stations is considered. This case is exactly the same as that of an elevator with stops at intermediate floors. It is shown how to find the characteristic times from figure 4-2 and that the capacity in vehicles per hour is found by dividing the value given in figure 4-2 corresponding to the distance between stops by $n - 1$, where n is the number of stations. If a bypass is placed at an intermediate station, as shown in figure 4-4, it is possible to run two vehicles on the shuttle and the capacity is doubled. If, however, the same idea is tried with two intermediate stations and four vehicles, the capacity does not double again but returns to the value for a simple shuttle of the same length. It is shown, therefore, that the advantage of including two or more intermediate bypass stations is not to increase capacity but to keep the capacity from reducing as the total line length increases.

In the next major section, section 4.2, the question of limitations on system capacity due to vehicles stopping at stations is considered. The criterion upon which the calculation is made is to keep the minimum distance between vehicles or trains greater than the required stopping distance if a failure should occur. For the case of a direct flow-through station, which may be either on-line or off-line, the results are summarized in a single dimensionless graph, figure 4-7. Here, the minimum time headway T can be found as a function of vehicle or train length L, station dwell time t_D, line speed V_L, normal and emergency deceleration, a and a_e, and the k-factor, where k is the ratio of the minimum distance between vehicles to the stopping distance of one vehicle. The case of end-of-the-line or back-up stations is then considered because this configuration is often used in line-haul systems to save space at the ends of the line. A dimensionless formula, equation (4.2.21), for the minimum time headway is given, and it is shown that, for the same line speed, the back-up station increases the minimum headway, but that, by reducing the line speed near the end stations, the headway possible at intermediate stations can be maintained.

Table 4-5 Classification of Transit Systems

	Shuttle	*Loop*	*Line-Haul*	*Network*
Stations:				
On-Line	A	A	A	A[a]
Off-Line	N/A	A	A	A
Lines:				
One-Way	A	A	N/A	A
Mixed	A	N/A	N/A	N/A
Two-Way	A	A	A	A
End Stations:				
Loop	N/A	N/A	A	N/A
Back-up	A	N/A	A	N/A
Service:				
Group	A	A	A	A[a]
Individual	N/A	A	A[b]	A

A—Applicable.
N/A—Not Applicable.
[a]In small networks only.
[b]Not for very high capacity.

The major section of the chapter, section 4.3, is devoted to loop systems. Here, basic performance equations are developed related not only to loops, but also to line-haul systems, which can be considered as collapsed loops, and to network systems, which comprise a multiplicity of connected loops. Six different types of loop systems, listed in table 4-5, are considered. Trip time matrices, demand matrices, and flow vectors are defined and it is shown how to find from them the average trip lengths and the required number of vehicles. The average trip length ratios, given in table 4-1, show that the capacity in a two-way system increases over that of a one-way system not only because there are two lines, but because of the decrease in average trip length. Thus, in comparing loops of say eleven stations, the capacity of a two-way system will be increased over that of a one-way system by a factor of $2(1.83) = 3.66$. With on-line stations, the required fleet size is the number of occupied vehicles required plus the extra vehicles needed in case of breakdowns. With off-line stations additional extra vehicles are needed to allow for redistribution of the vehicle fleet as a result of nonuniform demand. These are empty vehicles and the required number of them must be computed separately after the required number of occupied vehicles is found. Computation of the required number of occupied vehicles is summarized in table 4-2 for cases in which the average vehicle occupancy is known. For group systems, however, the number of occupied vehicles is found for a given schedule headway from equation (4.3.5) and then the average vehicle occupancy is found from the equations tabulated in table 4-2. The required number of empty vehicles is determined from the demand and trip time matrices and is computed most easily by the diagrammatic method shown in figure 4-10, because the assignment of empty vehicles is not unique in loop systems.

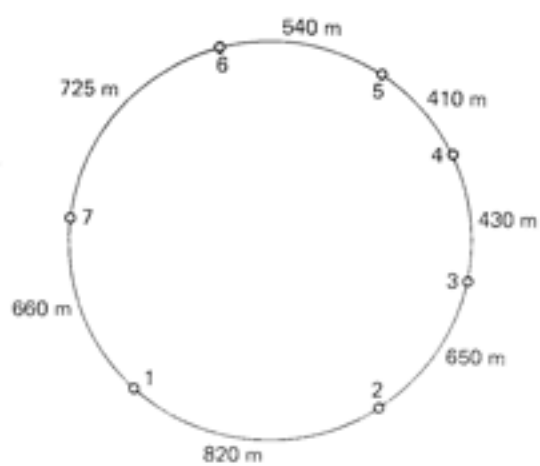
In section 4.4, the theory of loop systems is applied to line-haul systems, considered as collapsed loops, and minor variations needed for the case of line-haul systems are given. Finally, the theory of network systems is considered in section 4.5. First, a series of geometric performance parameters, including the average trip length, is derived for a square network but in a form in which they are approximately applicable to any network. Figure 4-18 shows how the average trip length approaches a limit value as the network size increases. Then, the performance parameters—fleet size, average time headway, average line and station flow, and nonstop wait time—are derived in a form applicable to networks. Of these parameters, the nonstop wait time bears comment: It is the time one would wait on the average for a second party headed for the same station. If this time is short, then group service nonstop between stations is practical. If it is long, nonstop group service is not practical and should be replaced by either a group service that permits stops at a number of intermediate stations, or by individual nonstop service. Since the nonstop wait time increases with the size of the network, the practical service policy for large networks is either nonstop on-demand or multistop scheduled. The difference in trip time in these two cases is found by subtracting equation (4.3.10) from equation (4.3.9), that is, it is the number of stops multiplied by the excess time. In practical cases, from equation (4.3.2), the excess time is in the range of thirty to forty seconds. A computation of nonstop wait time is given for a particular case in figure 5-6.

Problems

1. A simple shuttle is to be built to carry a maximum of 1500 people per hour per direction between two points 500 meters apart. The maximum cruise speed of the vehicle is 48 km/hr. Each vehicle has two doors, one through which people egress and the other through which they ingress. Four people per second can move through each door. If the vehicle is filled at peak loading, what is the required vehicle capacity?
2. An elevator service is to be provided for a 120-ft, 10-story building. The maximum flow rate for which the elevator system is to be designed is 500 people in 10 minutes during the morning or evening period in which people are traveling only in one direction. If each elevator makes an

average of four intermediate stops, dwelling at each floor for 5 seconds, how many elevators are needed? The maximum lift rate is 200 ft/min, the acceleration is 0.5 g, and the capacity of each elevator is 10 persons.

3. Develop an expression for the capacity of an on-line station loop system with unidirectional stations in ~~vehicles~~ cars per hour if the vehicles are coupled into n-car trains and the length of each car is L_c. Plot the capacity as a function of n in the range $1 \leq n \leq 10$ for (a) standing-passenger vehicles in which $a_e = a$, and (b) seated-passenger vehicles in which $a_e = 2a$. In both cases assume $k = 2$, $V_L = 25$ m/s, $L_c = 15$ m, and $t_D = 15$ s.
4. A heavy-rail system is used as a line-haul transit system with back-up end stations ($L_{max} = 95$ m) and eight-car trains. (a) Using the parameters of Problem 3 for standing-passenger vehicles, what is the capacity in people per hour if each car can hold 80 people and the average load factor is 60 percent. (b) By what percent is the capacity changed if the back-up end stations are replaced by loops, but the line speed around the loops must be reduced to 15 m/s? Did the speed reduction increase or decrease the capacity?
5. Assume that instead of coming directly into the station platform, the vehicles stop at a holding point where they wait for a platoon of n vehicles to load and leave together. The vehicles in the holding point then move forward together into the normal platform position. Determine the throughput of the station as a function of n and other pertinent kinematic parameters.
6. Consider the following loop systems. Distances between stations are given on the figure.



The demand matrix is as follows:

$$D_{ij} = \begin{bmatrix} 0 & 20 & 150 & 160 & 140 & 20 & 10 \\ 40 & 0 & 130 & 170 & 160 & 30 & 20 \\ 10 & 5 & 0 & 50 & 40 & 30 & 20 \\ 15 & 10 & 40 & 0 & 30 & 20 & 10 \\ 5 & 10 & 30 & 40 & 0 & 5 & 15 \\ 15 & 5 & 120 & 140 & 160 & 0 & 10 \\ 5 & 20 & 170 & 150 & 180 & 10 & 0 \end{bmatrix}$$

in which the units are people per hour.

Two different systems are to be considered, each in both one-way and two-way configurations:

singly

System 1: On-line stations, standing-passenger vehicles operating ~~in trains of two vehicles each~~ to increase reliability. The line speed is 15 m/s, ~~the vehicles can be assumed to be 10 m long~~, $k = 2$, and $a = a_e$.

System 2: Off-line stations, seated-passenger vehicles operating singly. The line speed is 10 m/s, the vehicles are 2.6 m long and accommodate 3 people. ~~The average load factor is~~ Assume 1.5 people per vehicle ~~and the dwell time is 5 seconds~~. Assume $k = 1$ and $a_e = 2a$.

Both types of vehicles can be loaded at a rate of ~~two~~ persons per second.

ambiguous... clarify

a. For one-way, counterclockwise flow, compute from D_{ij} the flow into and out of each station and the total demand.

b. Assuming the shortest length trip is always taken, separate the demand matrix into clockwise and counterclockwise components.

c. For two-way flow, compute the flow from the street into and out of each station, considering the platforms from which vehicles are boarded for travel in opposite directions to be separate stations. This information is used to size the stations.

d. Compute the total flow in each segment for one-way flow, and for two-way flow, on each track.

For System 1:

e. Write a formula for minimum headway, T, based on station throughput considerations. (t_D is expressed as a function of vehicle capacity C_v.)

f. For the one-way configuration, write a formula for C_v in terms of minimum headway, T, assuming full vehicles on the busiest segment. Solve this equation, together with the equation from e for C_v and T. Round C_v up to the nearest multiple of ~~five~~ two and call it the vehicle capacity, then round T up to the nearest multiple of 10 s and set the headway at this value. Compute t_D and round it up to the nearest multiple of 5 seconds. Compute the excess time T_{ex}.

g. Compute the circuit time and, as a matter of interest, the average speed. For the one-way configuration, compute the number of trains and the number of vehicles required.

h. For the one-way configuration, compute the average number of people per vehicle noting that it is the ratio of the average person-flow to the vehicle-flow.

i. For the two-way configuration, assume the same vehicle capacity as in the one-way system. Based on the flow of full vehicles in the busiest segment in each direction, compute the required headway in each direction. Compute the required number of two-vehicle trains and vehicles in each direction.

j. For the two-way configuration, compute the average number of people per vehicle in each direction.

For System 2:

k. Compute the excess time T_{ex}, and compute the average trip length for counterclockwise, one-way flow. Note that the corresponding matrix for clockwise flow is simply the transposed matrix.

l. Compute the average trip length for the two-way configuration in each direction.

m. Compute the number of occupied vehicles for the one-way and the two-way configuration.

n. Compute the excess-flow vector, EX_{vj}, for the one-way and the two-way configuration.

o. Draw a diagram for the dispatching of empty vehicles for the one-way and the two-way configuration, and compute the required number of empty vehicles in both cases.

p. Based on the total flow of vehicles on the busiest link, compute the minimum operating line headway for the one-way ~~or~~ the two-way configuration, and compute the minimum nose-to-tail spacing.

q. If the failure deceleration rate is twice the emergency deceleration rate, and the control time constant is 0.6 s, compute the ratio of minimum nose-to-tail spacing to the minimum no-collision spacing.

r. Compute the maximum station throughput in vehicles per hour, and with $p_v = 1.5$ compare it with the maximum required flow into a station in vehicles per hour, making certain to account for the flow of empty vehicles. If the requirement exceeds the maximum permissible throughput, the station must have more than one loading berth.

s. If both types of vehicles cost $3000 per unit capacity (see figure 5-1), compute the fleet cost of each of the four configurations. Recompute the fleet cost of the two versions of System 2 for a line speed of 15 m/s.

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