5 Cost Effectiveness

This chapter is divided into three parts. First, equations applicable to parametric analysis of the cost of any transit system are given to the level of detail in which the cost of vehicles, guideways, stations, and central facilities are each represented by lumped variables. Cost analysis of each of these types of equipment can fruitfully be carried out in much more detail in subsystem analysis. Some of this kind of analysis is indicated in later chapters; however, for systems analysis, the above categories of equipment carry the analysis to the required depth. Second, equations for analysis of cost effectiveness are given and discussed; and, third, the equations of cost effectiveness are applied to the analysis of specific types of systems. This work is based on the author's paper in the book *Personal Rapid Transit III*[1].

5.1 Cost Equations

The cost equations are given in the following list of notations following definition of the terms.

C_{vc} = vehicle cost per unit capacity. If all passengers are seated, C_{vc} is the cost per seat. If standees are allowed, C_{vc} is the vehicle cost divided by the design capacity, not the crushload capacity. The vehicle cost is denoted per unit capacity because, as shown by figure 5-1, the vehicle cost per unit capacity is not correlated with vehicle capacity.

q_c = vehicle design capacity

C_g = guideway cost per unit length. If the system uses two-way integral guideways, C_g is the cost of the two-way integral guideway. For convenience, C_g also includes the cost of right of way.

C_s = station cost including right of way, but not including off-line ramps

C_{sf_o} = portion of cost of support facilities not proportional to the number of vehicles

C_{sf_v} = portion of cost of support facilities proportional to the number of vehicles, per vehicle

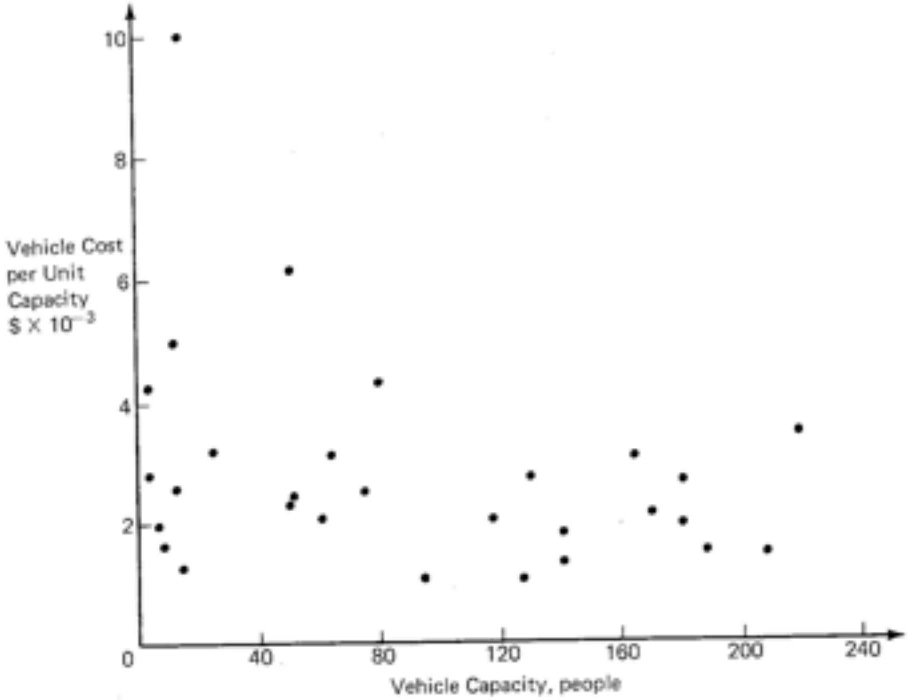

Figure 5-1. Guideway Transit Vehicle Cost per Unit Capacity (Data from 1975 Lea Transit Compendium)

l_r = length of an off-line ramp as determined by the theory of chapter 3

$\mathscr{L}$ = total guideway length. Given by equation (4.5.1) for network systems. If system uses two-way guideways, $\mathscr{L}$ is the total length of two-way guideways, not the one-way guideway length. Does not include off-line ramps.

n_s = total number of independent stations. Given by equation (4.5.4) for network systems.

N = the number of vehicles in the system. See equation (4.3.25).

Subscript o&m: This subscript is applied to the cost terms to denote the annual cost for operation and maintenance.

A_k = the amortization factor on the kth type of equipment, that is, the annual payment on the equipment for principal and interest divided by the initial cost. For convenience of readers not familiar with the economics literature, the formula for A_k is derived in Appendix A in terms of n_k, the life of the kth type of equipment, and r, the rate of interest on capital expenditure.

C_T = total initial cost of the system

$$C_T = (C_{vc}q_c + C_{sf_v})\,N + C_g\mathscr{L} + C_s n_s + C_{sf_o} \qquad (5.1.1)$$

C_{v_a} = annual cost of vehicles per vehicle

$$C_{v_a} = (A_v C_{vc} + C_{vc_{o\&m}})q_c \qquad (5.1.2)$$

C_{g_a} = annual cost of guideways per unit length

$$C_{g_a} = A_g C_g + C_{g_{o\&m}} \qquad (5.1.3)$$

C_{s_a} = annual cost of an average station

$$C_{s_a} = A_s C_s + C_{s_{o\&m}} \qquad (5.1.4)$$

C_{sf_a} = annual cost of support facilities

$$C_{sf_a} = A_{sf}(C_{sf_v}N + C_{sf_o}) + C_{sf_{o\&m}}N + C_{sf\ o\&m} \qquad (5.1.5)$$

C/yr = annual cost of system

$$C/\text{yr} = C_{v_a}N + C_{g_a}\mathscr{L} + C_{s_a}n_s + C_{sf_a} \qquad (5.1.6)$$

It is also useful to compute:

$(C/\text{yr})_{\text{cap}}$ = annual cost of amortization of capital equipment for entire system

$$(C/\text{yr})_{\text{cap}} = A_v C_{vc} q_c N + A_g C_g \mathscr{L} + A_s C_s n_s + A_{sf}(C_{sf_v}N + C_{sf_o}) \qquad (5.1.7)$$

$(C/\text{yr})_{o\&m}$ = annual system cost for operation and maintenance

$$(C/\text{yr})_{o\&m} = C_{vc_{o\&m}}q_c N + C_{g_{o\&m}}\mathscr{L} + C_{s_{o\&m}}n_s + C_{sf_{v_{o\&m}}}N + C_{sf_{o_{o\&m}}} \qquad (5.1.8)$$

5.2 Equations for Cost Effectiveness

Let t_d be the average number of trips per week day carried by the system.

Let t_{yr} be the average number of trips per year. Then in most cases

$$t_{yr} \approx 300\ t_d \tag{5.2.1}$$

The most basic cost effectiveness parameter is the *total cost per passenger trip* or the break-even fare. If this quantity is represented by C/tr,

$$C/\text{tr} = \frac{C/\text{yr}}{t_{yr}} \tag{5.2.2}$$

The *cost per vehicle trip,* $(C/\text{tr})_v$, is the cost per passenger trip multiplied by p_v. Then

$$(C/\text{tr})_v = (C/\text{tr})p_v \tag{5.2.3}$$

The *cost per passenger kilometer,* C/pkm, is C/tr divided by the average trip length $<L_t>$ in kilometers. Thus

$$C/\text{pkm} = \frac{C/\text{tr}}{<L_t>} \tag{5.2.4}$$

To determine the influence of freight hauling in addition to passenger hauling on the cost per passenger trip, let the number of freight trips per year, t_{yr_f}, be represented by

$$t_{yr_f} = et_{yr}/p_v \tag{5.2.5}$$

Thus e represents the ratio of vehicle trips for freight movement to vehicle trips for passenger movement. The total cost per year will be increased because of the need to provide for freight vehicles; however, if some of the passenger vehicles are used for freight movement in off-peak hours, the ratio of the number of freight vehicles to the number of passenger vehicles need not be as high as e. Let this ratio be

$$e' = \frac{N_{\text{freight}}}{N_{\text{pass}}} < e \tag{5.2.6}$$

Then, the cost per year as a function of the number of vehicles can be written in a form analogous to equation (5.1.6) in which N is multiplied by 1 + e' and n_s is increased if extra freight stations are added. As a first approximation, assume n_s is also multiplied by $1 + e'$. Then using equations (5.2.3), (5.2.2), (5.2.5), (5.2.6), (5.1.6) and (5.1.5),

$$C/\mathrm{tr} = \frac{(C/\mathrm{tr})_v}{p_v} = \frac{C/\mathrm{yr}}{P_v\left(\frac{t_{yr}}{p_v} + t_{yr_f}\right)} = \frac{C_1(1 + e') + C_2}{t_{yr}\ (1 + e\ )}$$

in which (5.2.7)

$$C_1 = (C_{v_a} + A_{ef}C_{sf_v} + C_{sf_{vo\&m}})N_p + C_{s_a}n_s$$

$$C_2 = C_{g_a}\mathscr{L} + A_{sf}C_{sf_o} + C_{sf_{oo\&m}}$$

where N_p is the number of vehicles needed for passenger service. With $e' \leq e$, it is clear that, because of the fixed facility costs, C_2, the addition of freight movement reduces the cost per passenger trip.

Consider an example. In a well-designed exclusive guideway system, $C_2 \approx C_1$. If, in the most extreme case, there are as many vehicle trips for freight movement as for passenger movement[2], $e = 1$. Finally, assume that half the freight trips are of such a nature that they can be handled by passenger vehicles in the off-peak hours. Then $e' = 0.5$. Substituting these three assumptions into equation (5.2.7) gives

$$C/\mathrm{tr} = \frac{2C_1}{t_{yr}}\left(\frac{5}{8}\right)$$

Thus, in this extreme case, the cost per passenger trip is reduced to 62.5 percent of its value if there is no freight movement. Freight movement is not considered in the derivation of the following cost effectiveness parameters, but it can be considered on the basis of the above analysis as the need arises.

The next cost effectiveness parameter, of interest to the transit operator, is the *annual surplus*, S_a, where

$$S_a = t_{yr}\ (\text{Average fare}) - (C/\mathrm{yr})_{o\&m} - (C/\mathrm{yr})_{cap} \qquad (5.2.8)$$

The two components of annual cost are broken out separately for emphasis because it is so common in contemporary transit studies to speak of a positive surplus when the annual revenue exceeds only the operation and maintenance costs. In capital intensive systems, $(C/\mathrm{yr})_{cap}$ exceeds $(C/\mathrm{yr})_{o\&m}$ often by a factor of more than two. Therefore, unless a system is under analysis in which the capital costs have been paid, it is not appropriate to refer only to an "operating surplus."

Another important cost effectiveness parameter is the *change in cost per trip if patronage is increased*. From equation (5.2.2), this is

$$\frac{\partial(C/\text{tr})}{\partial t_{\text{yr}}} = \frac{1}{t_{\text{yr}}}\left(\frac{\partial C/\text{yr}}{\partial t_{\text{yr}}} - \frac{C/\text{yr}}{t_{\text{yr}}}\right) \tag{5.2.9}$$

If t_{yr} is increased without adding fixed facilities, but only vehicles, then, since the vehicle fleet increases in proportion to t_{yr}, C/yr is of the form

$$C/\text{yr} = a_1 t_{\text{yr}} + a_2$$

where a_1 and a_2 are independent of t_{yr}. Then

$$\frac{\partial C/\text{tr}}{\partial t_{\text{yr}}} = -\frac{a_2}{t_{\text{yr}}^2}$$

and the cost per trip decreases as patronage is added because a_2 is greater than 0. If, to attract additional patronage, additional fixed facilities are built, then the situation may be different. If patronage is attracted in proportion to the cost of the new facilities, a_2 is proportional to t_{yr}, and C/tr is independent of t_{yr}. If costs of the fixed facilities increase more rapidly than in proportion to t_{yr}, then equation (5.2.9) shows that C/tr increases as t_{yr} increases and these new facilities must be defended on a basis other than direct cost. In general, equation (5.2.9) shows that if a curve of C/yr versus t_{yr} is drawn, the cost per trip will decrease as t_{yr} increases only if the slope of the curve is less than the slope of a line from the origin of coordinates to the point in question.

The final cost effectiveness parameter is the *present value of future savings* if the system in question is built rather than if present trends are continued. Let $(CS/\text{yr})_n^\circ$ represent the cost savings in the nth year in the future in base year currency if the new system is built. Then

$$\begin{aligned}(CS/\text{yr})_n^\circ = (C/\text{yr})^\circ_{\text{trend system}} &- (C/\text{yr})_{\text{new system}_{\text{cap}}} \\ &- (C/\text{yr})^\circ_{\text{new system}_{\text{o\&m}}}\end{aligned} \tag{5.2.10}$$

in which the cost per year of the new system is separated into the cost for capital and the cost for operation and maintenance. The yearly cost of the trend system and the operating and maintenance costs of the new system increase year by year due to inflation; but, once bonds are secured, the capital cost per year for principal and interest is fixed. If the inflation rate is

i per year, the cost saving in the nth year in the future in nth year dollars is

$$(CS/\mathrm{yr})_n^n = [(C/\mathrm{yr})^\circ_{\mathrm{trend\ system}} - (C/\mathrm{yr})^\circ_{\mathrm{new\ system}_{\mathrm{o\&m}}}](1 + i)^n - (C/\mathrm{yr})_{\mathrm{new\ system}_{\mathrm{cap}}} \tag{5.2.11}$$

Then, if d is the discount rate, the present value of the savings in the nth year is

$$(CS/\mathrm{yr})_{\mathrm{pv},n} = \frac{(CS/\mathrm{yr})_n^n}{(1 + d)^n} \tag{5.2.12}$$

From Equation (5.2.11), it is clear that, due to inflation, the cost savings increases year by year if a substantial portion of the system cost is in capital rather than in inflating costs. The cumulative present value of future savings out to the Nth year is

$$PV_N = \sum_{n=1}^{N} \frac{(CS/\mathrm{yr})_n^n}{(1 + d)^n} \tag{5.2.13}$$

If the cost terms in equation (5.2.11) are independent of n, the summation of equation (5.2.13) can be written in closed form using the identity

$$x + x^2 + x^3 + \dots + x^N = x(x^N - 1)/(x - 1)$$

Then, equation (5.2.13) becomes

$$PV_N = [(C/\mathrm{yr})^\circ_{\mathrm{trend\ system}} - (C/\mathrm{yr})^\circ_{\mathrm{new\ system}_{\mathrm{o\&m}}}] \times \left(\frac{1 + i}{i - d}\right)\left[\left(\frac{1 + i}{1 + d}\right)^N - 1\right] - (C/\mathrm{yr})_{\mathrm{new\ system}_{\mathrm{cap}}}\ 1/d\left[1 - \left(\frac{1}{1 + d}\right)^N\right] \tag{5.2.14}$$

Thus far, C/yr has included only the direct costs of the system. If the indirect costs due to factors such as air and noise pollution and land unavailable for other purposes are taken into account, as well as the direct cost to the traveler in terms of trip time, then PV_N becomes a true measure of the present value of the new system to society. This would seem to be a preferable measure of cost effectiveness of a new system to the more commonly used benefit/cost ratio because it quantifies the differences between new systems. Further usefulness of PV_N lies in the observation that, if the new system requires research and development to bring it into practical use, it is understandable that it would be justifiable to invest a small fraction of PV_N in research and development to realize the indicated cost savings.

5.3 Cost Effectiveness of Bus Systems

For bus systems that operate on surface streets, all of the annual costs are approximately proportional to the number of buses. Therefore let

$$C/\text{yr} = C_{b_a}N \tag{5.3.1}$$

in which C_{b_a} is the total annualized cost of the bus system for capital equipment, driver wages, and central facilities. In 1975, in the United States, C_{b_a} was approximately \$50,000, of which approximately 80 percent was driver wages.

If the minimum bus headway is given as $T_{\min}$, the number of buses is given by the following equation, analogous to equations (4.3.5):

$$N = \frac{2\mathscr{L}}{V_{av}T_{\min}} \tag{5.3.2}$$

Combining equations (5.3.1), (5.3.2), (5.2.2) and (5.2.1),

$$C/\text{tr} = \left(\frac{C_{b_a}\mathscr{L}}{150V_{av}T_{\min}}\right)\frac{1}{t_d} \tag{5.3.3}$$

If the bus system is a network of lines, define the daily trip density $\bar{t}_d$ by the equation

$$\bar{t}_d = \frac{t_d}{A} \tag{5.3.4}$$

in which A is the area covered by bus lines. Then, substituting equations (4.5.1) and (5.3.4) into equation (5.3.3),

$$C/\mathrm{tr} = \left(\frac{C_{b_a}\beta}{75 V_{\mathrm{av}} T_{\mathrm{min}} L} \right) \frac{1}{\bar{t}_d} \qquad (5.3.5)$$

Consider a typical example of a large bus network for which $\beta = 1$, $V_{\mathrm{av}} T_{\mathrm{min}} = 1$ mi $= 1.6$ km, and $L = 0.5$ mi (0.8 km). (This case corresponds, for example, to $V_{\mathrm{av}} = 10$ mi/h and $T_{\mathrm{min}} = 6$ minutes.) Substituting these values and $C_{b_a} = \$50{,}000$, into equation (5.3.5),

$$C/\mathrm{tr} = (13.3¢)\ \frac{10^4}{\bar{t}_d} \qquad (5.3.6)$$

Equations (5.3.5) and (5.3.6) apply for values of t_d up to the point of saturation, that is, up to the point where more trips can be handled only by adding more buses. If the bus system is saturated, N must be determined by equation (4.5.17) in which p_v is the saturation value of the average number of people per bus. Then, setting $f_p = 1$ and letting

$$T_{\mathrm{ex}} + \frac{\langle L_t \rangle}{V_L} = \frac{\langle L_t \rangle}{V_{\mathrm{av}}}$$

$$N = \frac{\sigma \bar{t}_h \langle L_t \rangle A}{p_v V_{\mathrm{av}}} \qquad (5.3.7)$$

Substitute equation (5.3.7) into equation (5.3.1), and then equations (5.3.1), (5.3.4) and (5.2.1) into equation (5.2.2) to give

$$C/\mathrm{tr} = \frac{C_{b_a} \sigma \langle L_t \rangle}{3000 p_v V_{\mathrm{av}}} \qquad (5.3.8)$$

and the headway corresponding to p_v is found by equating equations (5.3.2) and (5.3.7), with equation (4.5.1) substituted. Thus

$$T_{\mathrm{min}} = \frac{40 \beta p_v}{\sigma \bar{t}_d L \langle L_t \rangle} \qquad (5.3.9)$$

in which it has been assumed that $\bar{t}_d = 10\bar{t}_h$. Assuming $C_{b_a} = \$50{,}000$, $\sigma = 1.05$, and $V_{av} = 16$ km/hr, equation (5.3.8) becomes

$$C/\text{tr} = \$1.75\,\frac{<L_t>}{p_v} \tag{5.3.10}$$

in which $<L_t>$ is in kilometers. Equating equation (5.3.10) and (5.3.6), it is seen that saturation of the bus system occurs when

$$\bar{t}_d = 760\,\frac{p_v}{<L_t>} \tag{5.3.11}$$

Equation (5.3.6) and equation (5.3.10) for several values of T_{min}, $<L_t>$, and p_v are plotted in figure 5-2.

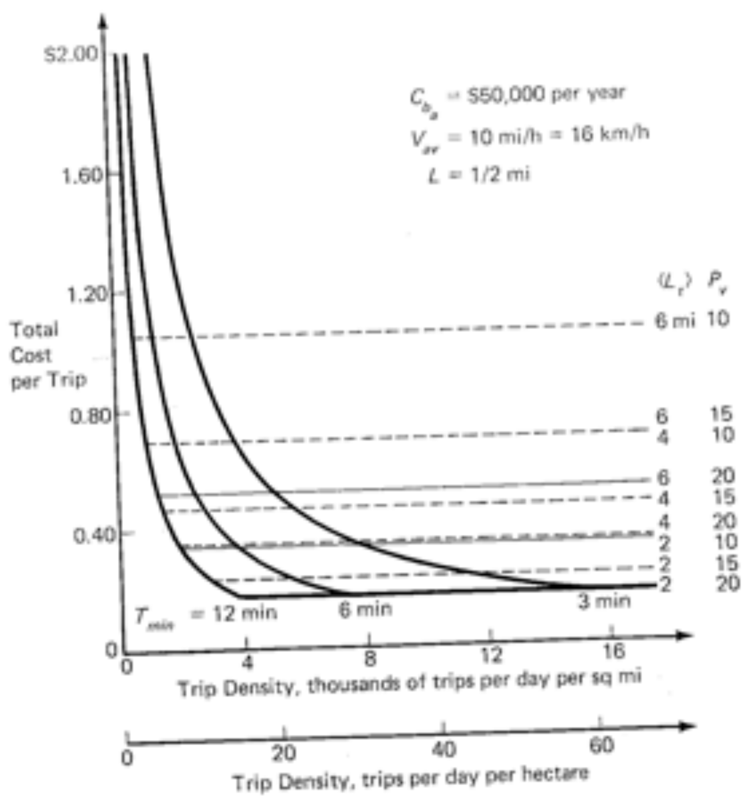

Figure 5-2. Total Cost per Trip of Bus Systems

Similar analyses can be carried through for the other cost effectiveness parameters, but for bus systems that does not seem worthwhile at this point. Understanding of the behavior of C/tr with trip density, trip length, minimum headway, and the saturation number of people per bus gives a good understanding of the cost effectiveness of bus systems.

The trip density can be interpreted by noting that

$$\bar{t}_d = m_t \, \tau_d \rho \tag{5.3.12}$$

in which ρ is the number of people per square mile, τ_d is the mobility, that is, the total number of trips per person per day, and m_t is the fraction of the number of daily trips taken by bus transit. If the bus network covers only a portion of the metropolitan area, $\bar{t}_d$ is composed of three types of trips:

1. Trips internal to the network
2. Trips from points outside the network to points inside
3. Trips from points inside to points outside

Analysis of this kind of trip distribution pattern is deferred to the next chapter.

5.4 Cost Effectiveness of Shuttles

In analysis of cost effectiveness of shuttles, the cost per vehicle trip is the most appropriate parameter. Combining equations (5.2.1) through (5.2.3),

$$(C/\mathrm{tr})_v = \frac{C/\mathrm{yr}}{300 t_d / p_v} \tag{5.4.1}$$

In the case of a simple shuttle, $N = 1$ and $n_s = 2$. Therefore equation (5.1.6) can be written

$$C/\mathrm{yr} = C_{vssf_a} + C_{g_a} \mathscr{L} \tag{5.4.2}$$

in which

$$C_{vssf_a} = C_{v_a} + 2C_{s_a} + C_{sf_a} \tag{5.4.3}$$

It is convenient to express t_d/p_v in terms of capacity. The capacity of a shuttle is given by equation (4.1.4) in which $D_s = \mathscr{L}$ and the velocity and times are given in seconds. Let

$$T_{ex} = t_D + \frac{V_L}{a_m} + 1 \tag{5.4.4}$$

~~and~~
Then

$$t_d/p_v = 10\alpha\left(\frac{1800V_L}{\mathscr{L} + V_L T_{ex}}\right) \tag{5.4.5}$$

in which α is a factor between zero and one, and it is assumed that the daily number of vehicle trips is ten times the number of peak-hour vehicle trips. Substituting equations (5.4.2) and (5.4.5) into equation (5.4.1),

$$(C/\mathrm{tr})_v = C_0(1 + C_1\mathscr{L})\,(1 + C_2\mathscr{L}) \tag{5.4.6}$$

in which

$$C_0 = C_{vssf_a}T_{ex}/5.4(10)^6\alpha$$

$$C_1 = C_{g_a}/C_{vssf_a}$$

$$C_2 = 1/V_L T_{ex}$$

Thus, it is seen that the cost per vehicle trip for a shuttle is a quadratic function of the length of the shuttle. To give the reader a feeling for the cost per vehicle of a typical shuttle, consider the following example:

$$C_v = C_{v_c}q_c = \$80{,}000$$

$$C_s = \$100{,}000$$

$$C_g = \$1000/\mathrm{m}$$

$$C_{sf} = \$50{,}000$$

Let the vehicles be amortized over an assumed life time of fifteen years and the fixed facilities over forty years, all at an interest rate of 6 percent. Then, from Appendix A, $A_v = 0.103$, and $A_g = A_s = A_{sf} = 0.066$. Let the annual operating and maintenance costs for the vehicle be 5 percent of the capital cost and for the stationary equipment be 2 percent of the capital cost. Then, from equations (5.1.2 through 5.1.5), and equation (5.4.3),

$$C_{v_a} = \$12{,}240 \qquad C_{g_a} = \$86/\mathrm{m}$$

$$C_{s_a} = \$8600 \qquad C_{sf_a} = \$4300 \qquad C_{vssf_a} = \$33{,}740$$

Let $T_{ex} = 30$ s and $V_L = 10$ m/s. Then

$$C_0 = 0.187/\alpha \qquad C_1 = 0.00255 \qquad C_2 = 0.00333$$

Equation (5.4.6) is plotted in figure 5-3 for several values of T_{ex} and V_L, and for $\alpha = 1$. Thus, once the flow per day is determined as a fraction of capacity per day, the cost per trip may be found by dividing the values from figure 5-3 by α. The costs used are representative only, and computations made for specific cases should be based on manufacturer's data. The curves terminate at the low end at the minimum length for which the indicated line velocity is attainable at an acceleration of 1.25 m/s^2. (See the sentence below equation (2.4.6).)

5.5 Cost Effectiveness of Loop Systems

The number of occupied vehicles required in a loop system is given by table 4-2. In the present analysis, it is convenient to use the average velocity V_{av}, defined for loops by the equation

$$V_{av} = \frac{\langle L_t \rangle}{\gamma T_{ex} + \langle L_t \rangle / V_L} \tag{5.5.1}$$

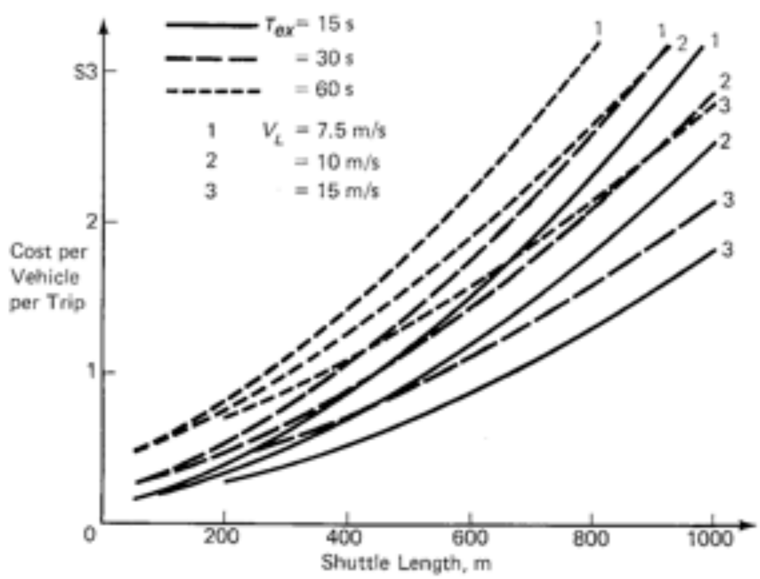

Figure 5-3. The Cost per Vehicle Trip of a Typical Shuttle (operating at capacity 10 hours per day 300 days per year)

in which the equations that give γ and $<L_t>$ are listed in table 4-2. Then, using the definitions given by equations (4.5.14) and (4.5.15), the total number of vehicles is

$$N = \frac{\sigma <L_t> t_d / 10}{f_p p_v V_{av}} \tag{5.5.2}$$

in which $t_d = 10\,\bar{D}_{peak}$ is the assumed daily travel. Equation (5.5.2) is applicable until the minimum headway, given by equation (4.3.6), is reached. If t_d is increased further, training or off-line stations must be used.

Using equations (5.1.6) and the definitions that lead to it, and (5.2.1), equation (5.2.2) becomes

$$C/\mathrm{tr} = \frac{1}{300 t_d}\left[C_{c_a} q_c N + C_{g_a}(\mathscr{L} + 2 l_r \mu n_s) + C_{s_a} n_s + C_{sf_{o_a}}\right] \tag{5.5.3}$$

in which

$$C_{c_a} = A_v C_{vc} + C_{vc_{o\&m}} + (A_{sf} C_{sf_v} + C_{sf_{v\,o\&m}})/q_c$$

$$C_{sf_{o_a}} = A_{sf} C_{cf_o} + C_{sf_{o\&m}}$$

μ = ratio of cost of curved guideway to cost of straight guideway

and the term $2 l_r \mu n_s$ is added to the guideway length to explicitly account for off-line stations. If the stations are on line, this term is dropped and $f_p = 1$.

Substituting equation (5.5.2), equation (5.5.3) becomes

$$C/\mathrm{tr} = \frac{C_{c_a} \sigma <L_t>}{3000 f_p V_{av} (p_v/q_c)} + \frac{C_{g_a} \mathscr{L} + C_{sf_{o_a}}}{300 t_d} + \frac{(2 C_{g_a} l_r \mu + C_{s_a}) n_s}{300 t_d} \tag{5.5.4}$$

in which p_v/q_c is the average vehicle load factor.

As indicated in the derivation of C_{v_c}, C_{v_c} and, for similar reasons, C_{c_a} are very weakly correlated with vehicle capacity. Hence, the first term in equation (5.5.4) depends on vehicle capacity directly only if vehicle size influences average speed and load factor. In on-line station systems, $f_p = 1$, but the intermediate stops lower V_{av} thus raising the vehicle cost component of the cost per trip. Larger vehicles must wait longer at stations to increase the load factor, thus reducing V_{av} while attempting to increase (p_v/q_c). If the stations are off-line, f_p may reduce to about two thirds but V_{av} increases substantially, for given V_L, due to elimination of intermediate stops (see figure 2-4). Also, as vehicle capacity decreases, the station dwell time required to obtain a significant daily average load factor decreases, thus increasing both V_{av} and p_v/q_c. If the service is on demand such that the vehicle leaves the station with only one party aboard, it is apparent that the first term in equation (5.5.4) is minimized. If the guideway is made bidirectional by permitting flow in opposite directions either side-by-side or above and below the guideway, the average trip length is substantially reduced, as indicated in table 4-1, thus reducing the vehicle cost term in equation (5.5.4).

The numerator of the third term in equation (5.5.4) increases due to addition of off-line stations because of the addition of off-line ramps. However, the station platform itself is generally shorter with off-line stations, thus reducing C_{s_g}. Moreover, the increased average trip speed, V_{av}, possible with off-line stations generally increases ~~to~~ t_d. Thus, the direction of the net change in C/tr due to addition of off-line stations requires detailed analysis of a range of specific examples.

For a given route length, $\mathscr{L}$, the second term in equation (5.5.4) depends mainly on C_{g_g} and t_d. The guideway cost per unit length, C_{g_g}, depends on three factors:

1. The weight per unit length of the vehicles
2. The cross sectional dimensions of the guideway
3. The maximum speed

In figure 5-4, the weights per unit length of operational and developmental transit vehicles are plotted as a function of design capacity. Lower weight per unit length of the vehicles permits reduction in guideway weight per unit length and hence in guideway cost. An even more effective way to minimize guideway cost, however, is to choose the guideway cross-sectional shape so as to minimize guideway weight per unit length. This subject is discussed in chapter 10.

The influence of maximum cruising speed on guideway cost is indicated by equation (3.6.21) which shows that, for a given guideway misalignment, the maximum lateral jerk is proportional to the cube of the speed. Thus, for specified maximum lateral jerk, the misalignment tolerances increase very rapidly with speed, thus requiring a more rigid, more accurately aligned, and hence more expensive guideway to accommodate higher speeds. In this regard, the comparison between on-line and off-line stations is signifi-

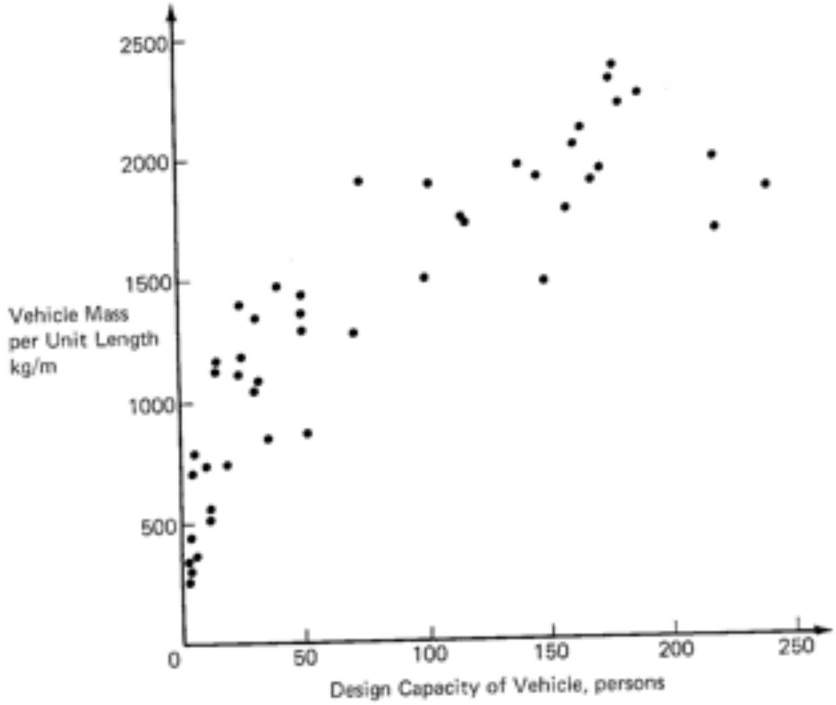

Figure 5-4. Transit Vehicle Mass per Unit Length
Source: 1975 Lea Transit Compendium

cant. With off-line stations, figure 2-4 shows that V_{av}/V_L is much closer to unity than with on-line stations. In typical cases, this ratio is in the neighborhood of 0.6 for on-line station systems and 0.95 for off-line station systems. Thus, for a given average speed, the maximum speed is considerably lower if off-line stations are used.

Finally, the patronage term t_d in the second and third terms of equation (5.5.4) is greatest and hence the guideway cost portion of C/tr least if the trip time is minimum, that is, if V_{av} is maximum. Thus, high V_{av} lowers all terms of equation (5.5.4). However, if V_{av} is increased by increasing V_L, C_{g_a} increases, as indicated above, and also C_{c_a} increases because higher V_L requires higher motor power, approximately in proportion to V_L^2. Also, l_r increases with V_L, as indicated by the theory of sections 3.4 and 3.5 Thus, there is a value of V_L that minimizes C/tr. Based upon detailed parametric analysis of the cost of guideways and vehicles, and of the dependence of patronage on V_{av}, the optimum value of V_L can be determined in specific cases. Determination of the optimum V_L for a range of practical cases is of fundamental importance in the cost effective design of guideway transit systems.

5.6 Cost Effectiveness of Line-Haul Systems

Since a line-haul system is a collapsed loop, the analysis of cost effectiveness follows the line of argument developed in section 5.5. The headway limitation is determined by the analysis of section 4.2 and may be different if the end stations are reversing as shown in figure 4-8 rather than if they permit unidirectional flow, as shown in figure 4-6. Equation (5.5.4) is used to compute the cost per trip, in which, for line-haul systems, $\mathscr{L}$ is the length of two-way lines and C_{g_a} is the annual cost per unit length of two-way lines.

In section 5.5, the terms of equation (5.5.4) were analyzed qualitatively to determine the variation of C/tr with various design options. Here, we will place some numerical estimates on the parameters in each of the three terms of equation (5.5.4). The costs assumed will be typical of several types of line-haul systems, and the purpose of the analysis is to obtain a feeling for the magnitudes and the ranges of variables needed to make the system economically feasible. Much actual cost data can be obtained from the Lea Transit Compendium[3] for specific systems of all types. However, to avoid reference to the equipment of specific manufacturers, the numbers assumed here must be considered representative only.

Consider the first term in equation (5.5.4), the vehicle cost per trip. From figure 5-1, a representative value of vehicle cost per unit capacity is about \$2500 per unit capacity. Assume that the amortization factors are as computed in section 5.4, that the annual vehicle cost for operation and maintenance is 5 percent of the capital cost, and that the annual cost for support facilities is 30 percent of the annual vehicle cost. Then, from the definition below equation (5.5.3), C_{c_a} = \$500 per unit capacity per year. Consider an on-line station system. Then $f_p = 1$ and in typical modern cases $V_{av} = 60$ km/hr. Let $\sigma = 1.05$. Then

$$(C/\mathrm{tr})_{\mathrm{vehicles}} = \$0.003\ \frac{\langle L_t \rangle}{p_v/q_c}$$

In line-haul systems $\langle L_t \rangle = 8$ km is representative. The number of people per vehicle, p_v, comes from equation (5.5.2) and must be representative of rush period values. Assume $p_v/q_c = 0.2$. Then

$$(C/\mathrm{tr})_{\mathrm{vehicles}} = \$0.12 \text{ (driverless vehicles)}$$

If each vehicle has a driver, add \$30,000/q_c year to $C_{vc_{o\&m}}$. For typical train systems, assume the vehicle design capacity is 100 people per vehicle. Thus C_{c_a} = \$800 and

$$(C/\mathrm{tr})_{\mathrm{vehicles}} = \$0.19 \text{ (driven vehicles)}$$

In this hypothetical case, there is a savings of 7¢ per trip by use of automatic control if the vehicles are large. With $q_c = 10$, the savings would have been $0.84 − 0.12 or 72¢ per trip. Thus there is a substantial advantage in going to automatic control only if small vehicles are contemplated. The actual savings is smaller than indicated if account is taken of the increased cost per vehicle due to automatic control equipment.

While the vehicle cost term in equation (5.5.4) appears to be independent of the patronage, t_d, equation (5.5.2) shows that p_v declines in proportion to t_d with vehicles operating at a fixed rush period headway. Thus, if t_d falls below the value used to compute N, in which computation p_v is assumed to be a reasonable fraction of q_c, $(C/\text{tr})_{\text{vehicle}}$ rises because p_v falls.

In the above estimations, V_{av} was assumed to be 60 km/hr = 16.7 m/s. From equation (2.4.4) or figure 2-4, such a high average speed can be obtained only with wide station spacing and high line speed. For example, if in the rush period the dwell time averages 40 s, a_m = 1.25 m/s², and the station spacing is 2.4 km (1.5 mi), V_{av} = 16.6 m/s if V_L = 30 m/s. If increased access is desired by placing stops say one half mile or 0.8 km apart, then a V_L of 30 m/s can still be achieved but this results in an average speed of only 8.73 m/s (19.6 mi/h). Thus, the values of $(C/\text{tr})_{\text{vehicle}}$ computed above must be multiplied by the ratio 16.6/8.73 = 1.9.

If the system under consideration is a street car with stops every quarter mile or 0.4 km, the maximum achievable speed at a_m = 1.25 m/s and a_m/J = 1 is (see equation (2.4.5)) V_L = 21.7 m/s = 48.9 mi/h. This is too high a maximum speed for street service. Assume instead V_L = 35 mi/h = 15.6 m/s. Then, from the same conditions, equation (2.4.4) gives V_{av} = 5.06 m/s (11.4 mi/h). If t_D is reduced to 10 seconds, V_{av} = 8.14 m/s (18.3 mi/h). Thus, in these cases, the vehicle cost per trip is increased by factors of 3.30 and 2.05, respectively.

If off-line stations are used in the same example with a trip length of five miles, D_s = 8 km in equation (2.4.4), and with a_m = 1.25 m/s², t_D = 40 s, and V_L = 17 m/s, V_{av} = 15.2 m/s. Thus, the average speed is only 10 percent below line speed. If a_m = 2.5 m/s², assuming seated passengers, and t_D = 10 s, V_{av} = 16.4 m/s or only 4 percent below line speed. As indicated in section 5.5, by obtaining an average speed only slightly below the line speed, the vehicle cost per trip can be kept low while not penalizing the guideway cost per trip by having to design for an excessively high maximum speed.

In estimating typical levels of the second and third terms of equation (5.5.4), it is necessary to develop a simple model for estimation of t_d. Thus, assume a line-haul system draws patronage from an area of length $\mathscr{L} + W$ and width W. Then, combining equations (5.3.4) and (5.3.12),

$$t_d = m_t \tau_d \rho A = m_t \tau_d \rho W(\mathscr{L} + W) \qquad (5.6.1)$$

In a typical case, assume $W = 2$ mi and $\mathscr{L} = 10$ mi. In typical U.S. urban areas, τ_d is roughly three trips per person per day. Assume a nominal case in which $\rho = 10{,}000$ people per sq mi and $m_t = 0.05$. Then $t_d = 36{,}000$ trips per day. This is typical of the trip attraction of rail rapid transit systems in the United States[4].

For elevated rail systems C_g is in the range of \$10 million to \$20 million per mile. For subways, the cost rises to the range of \$40 million per mile, and for surface systems, it may be as low as \$2 million per mile. Assume, as in section 5.4, $A_g = 0.066$ and $C_{g_{o\&m}}/C_g = 0.02$. Then, from equation (5.1.3), $C_{g_a} = 0.086\ C_g$. For convenience in this estimation, assume $C_{sf_{o_a}} = 0.2 C_{g_a}\mathscr{L}$. Then the guideway cost per trip term in equation (5.5.4) is

$$
\begin{aligned}
(C/\mathrm{tr})_{\mathrm{guideway}} &= \frac{C_{g_a}\mathscr{L} + C_{sf_{o_a}}}{300 t_d} \\
&= \frac{0.103 C_g \mathscr{L}}{300 t_d} \qquad (5.6.2)
\end{aligned}
$$

Substituting for t_d from equation (5.6.1) and then the numerical parameters listed under that equation,

$$(C/\mathrm{tr})_{\mathrm{guideway}} = 0.095(10)^{-6} C_g$$

Thus, if $C_g = \$2(10)^6$, $(C/\mathrm{tr})_{\mathrm{guideway}} = 19¢$ per trip, and it is clear that even with modest guideway cost, the component of cost per trip due to the guideway is well above the component due to the vehicles. If a twenty-million-dollar-per-mile guideway is used, it can be justified only if the patronage is substantially higher. From equation (5.6.1), patronage can be increased by increasing the mode split m_t, by considering such a system only in very high population density corridors, or by increasing the area coverage. Assuming V_{av} is already as high as practical, m_t can be increased only by improving access to the system by drawing from a larger area. However, many studies of rapid rail including access modes indicate that, in most communities, a daily mode split of even 10 percent is highly optimistic[4]. With on-line stations, attempts to increase access and hence m_t by placing the stations close together result in lowered V_{av} and hence the sought-after increase in m_t is not impressive. V_{av} can be kept high and m_t at a maximum only with off-line stations, and nonstop, on-demand service. Even then, if the system only serves a narrow corridor and not an area, the expected increase in m_t is generally not impressive. From the analysis of $(C/\mathrm{tr})_{\mathrm{guideway}}$ it seems clear that the promise of guideway transit lies in keeping C_g under \$2 million per mile and m_t as high as possible by providing minimum trip-time service.

Consider the station contribution to cost per trip. From equations (3.4.3) and (2.2.6), the length of an off-line ramp into off-line station is approximately

$$l_r = V_L \left[\left(\frac{32H}{J_n} \right)^{1/3} + \frac{V_L}{2a_m} + \frac{a_m}{2J} \right] \qquad (5.6.3)$$

For off-line station systems, assume for the present analysis that $a_m = J = J_n = 2.5$ m/s², $V_L = 15$ m/s, and $H = 2.5$ m. Then $l_r = 100$ m. For these systems, also assume that $\mu = 1.2$ and $C_g = \$2(10)^6$ per one-way mile or \$1250 per meter. Thus $C_{g_a} = 0.086$ (\$1250) $=$ \$107.50/m and $2\ C_{g_a} l_r \mu =$ \$25,800 per year. For off-line station, small-vehicle systems, C_s has been estimated in the range of \$100,000. Thus, using the same amortization factor and ratio of capital to operating and maintenance costs as for guideways, $C_{s_a} = \$8600$ per year. Thus, for off-line station systems, we estimate the total station cost per year as

$$2\ C_{g_a} l_r \mu + C_{s_a} = \$34{,}400 \text{ per year}$$

But, for a two-way line-haul system, each "station" is two one-way stations with a cost of \$68,800 per year. With $\mathscr{L} = 16$ km (10 mi), we estimated $C_{g_a}\mathscr{L} + C_{sf_{o_a}} = 0.103\, C_g \mathscr{L}$, or with $C_g = \$4(10)^6$ per two-way mile $\times$ 10 miles, $C_{g_a}\mathscr{L} + C_{sf_{o_a}} = \$4.12(10)^6$ per year. If there is one station per mile, $n_s = 11$, and the total annual cost for stations if \$68,800 (11) $=$ \$757,000 or 18 percent of the guideway cost. If the stations are half a mile apart, their annual cost in this example is 35 percent of the guideway cost. Usually, the cost per two-way mile of guideway is not twice the cost of one-way guideway because of economies in placing two guideways on a single set of supports, but in the range of 30 percent less per unit length. If this is the case, the station cost is 26 percent and 50 percent of the guideway cost, respectively, in the above example.

If the stations are on-line, the platforms are generally larger and the structure larger. Costs of rail rapid transit stations are quoted in the range of \$500,000 to \$1 million and higher, which exceed the cost of off-line stations counting the off-line ramps. For this reason, and because of high guideway cost, the so-called "light rail" transit option is often considered. It is attractive if it does not require exclusive right of way, if the track can be conveniently laid at surface streets, and if enclosed stations are not needed. In these cases, the components of cost due to guideways and stations become manageable. Unfortunately, however, lower cost ways and stations usually mean interference with street traffic and hence reduced average speed, which increases the vehicle component of cost per trip.

If the line-haul system consists of forty-passenger buses operating in mixed traffic or on freeways, the major cost is the vehicle cost. Then, in the above example, the driver cost term in the cost per year is in the neighborhood of \$30,000/40 = \$750 and $(C/\mathrm{tr})_{\mathrm{vehicles}}$ is approximately 21 ¢ per trip, if V_{av} = 60 km/hr, and rises in inverse proportion to V_{av}.

Again, it must be emphasized that the above calculations are representative only, and that conclusions for policy purposes should be based on analysis of specific situations. By following the above analysis, however, the reader can quickly estimate the cost per trip in specific cases. The other cost effectiveness variables derived in section 5.2 can be computed readily once the cost per trip and number of trips per year are known, and these need no further elaboration here.

5.7 Cost Effectiveness of Guideway Network Systems

In section 5.3, the cost per trip of network bus systems was discussed. Here, a similar analysis is carried forward for network systems in which automated vehicles run on exclusive guideways. For the analysis of network systems, equation (5.5.4) is still the basis, except that an additional term must be added to account for extra ramps at interchanges. Equation (4.5.6) gives a formula for the number of intersections in a network system.

Figure 5-5 shows two basic types of interchanges: the multilevel interchange and the Y-interchange. Both permit two perpendicular streams to go straight or turn through the interchange. The multilevel interchange has the advantages that traffic streams diverge before they merge, and both

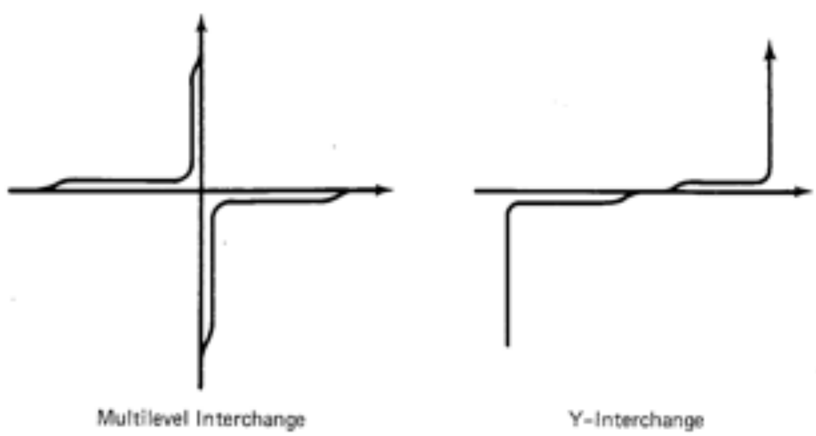

Figure 5-5. Network Interchanges

streams going straight through do not have to turn. It has the disadvantages, however, that the through guideways have to be at different levels, and the visual impact of guideways at one location may not be acceptable. The Y-interchange has the advantages of being all at one level and of minimum visual impact, but the disadvantages that the traffic streams must merge before they diverse, thus doubling the flow on the line through the interchange, and that the traffic on one of the lines must make unwanted turns through the interchange. In the cost analysis the difference is that the multilevel interchange uses four ramps and the Y-interchange two. Thus, define an interchange factor Z, where Z is equal to 1 for Y-interchanges and 2 for multilevel interchanges. Therefore, equation (5.5.4) becomes

$$C/\mathrm{tr} = \frac{1}{300}\left\{ \frac{C_{c_a}\sigma \langle L_t \rangle}{10 f_p V_{av}(p_v/q_c)} + \frac{1}{t_d}\left[C_{g_a}\mathscr{L} + (2C_{g_a} l_r \mu + C_{s_a})n_s + 2Z l_r \mu C_{g_a} n_I + C_{sf_{o_a}} \right] \right\} \tag{5.7.1}$$

The network values for $\mathscr{L}$, n_s, and n_I are given by equations (4.5.1, 4.5.4, and 4.5.6), respectively; and, from figure 4-18, let

$$\langle L_{av} \rangle = \nu A^{1/2} \tag{5.7.2}$$

Using these equations and equation (5.3.4), equation (5.7.1) becomes

$$C/\mathrm{tr} = \frac{1}{300}\left\{ \frac{C_{c_a}\sigma \nu A^{1/2}}{10 f_p V_{av}(p_v/q_c)} + \frac{(2\beta/L)[C_{g_a} + (2 + Z\beta)\mu C_{g_a} l_r/L + C_{s_a}/L] + C_{sf_{o_a}}/A}{t_d} \right\} \tag{5.7.3}$$

To give a feeling for magnitudes, consider a specific example. A typical automated system suitable for network operation and for which cost data [5] is available is the Cabintaxi system under development since 1970 by DEMAG Fördertechnik and Messerschmitt-Bölkow-Blohm GmbH.

The parameters for this system are as follows:

q_c	=	3	σ	=	1.03
C_{ca}	=	\$1450	μ	=	1.2
C_{ga}	=	\$125/m	f_p	=	2/3
C_{sa}	=	\$5590	V_{av}	=	10 m/s
C_{sfo_a}	=	\$180,000	l_r	=	90 m
Z	=	1	p_v/q_c	=	0.5

In addition let the line spacing be $L = 800$ m. Then consider the two network sizes depicted in figures 4-16 and 4-17. Thus, for:

Figure 4-16

$$A^{1/2} = 3L = 2400 \text{ m}$$
$$\beta = 4/3$$
$$\nu = 1.21 \text{ for two-way flow (fig. 4-18)}$$
$$= 1.44 \text{ for one-way flow}$$

Figure 4-17

$$A^{1/2} = 5L = 4000 \text{ m}$$
$$\beta = 1.2$$
$$\nu = 0.99 \text{ for two-way flow (fig. 4-18)}$$
$$= 1.21 \text{ for one-way flow}$$

The costs given above are for two-way guideways and stations, with vehicles running above and below the guideway. With one-way guideways and stations, the cost of these facilities, in the Cabintaxi system, is reduced by about 25 percent.

The quantity 10 in the first term of equation (5.7.3) is approximate and has units of hours per day. Therefore, with V_{av} in meters per second, the first term must be divided by 3600 seconds per hour. The quantity $\bar{t}_d$ in equation (5.7.3) is not a true trip density because, in its definition given by equation (5.3.4), it is divided by the area bordered by the guideway. If, however, $\bar{t}_d$ is broken down into components, as indicated by equation (5.3.12), it is usual to think of ρ as the average number of people per unit area within the area served by the network area. Call this area A'. Then, for the network of figure 4-16, assume that $A' = (4L)^2$; and for figure 4-17, $A' = (6L)^2$, that is, $A'/A = (4/3)^2$ and $(6/5)^2$, respectively. Now, to be able to consider $\bar{t}_d$ in equation (5.7.3) as a true trip density, multiply $\bar{t}_d$ by A'/A.

With those modifications, equation (5.7.3) can be written in the form

$$C/\text{tr} = C_1 + C_2/\bar{t}_d \tag{5.7.4}$$

Values of C_1 and C_2 together with key geometric and performance parameters are given in table 5-1. In the table, it is assumed that the units of $\bar{t}_d$ are trips per day per hectare (1 h = 10^4 m^2, 1 sq mi = 259 h).

The performance parameters for the data of table 5-1 are plotted in figure 5-6 as functions of trip density. The curves labelled "S" correspond to the network shown in figure 4-16 for a line spacing of 800 meters or one half mile, and the curves labelled "L" correspond to the network shown in figure 4-17 also for a line spacing of 800 meters. Data are plotted for each of these networks for both one-way and two-way lines.

In the upper graph of figure 5-6, the lines proportional to trip density give the fleet size. In each network more vehicles are required if the lines are one way because the trip length is longer in that case. In the two-way network, half of the vehicles are on each side of the guideway.

The average headway is derived from equation (4.5.18) except that for two-way lines, T_{av} is doubled because half of the vehicles are on each side

Table 5-1 Geometric, Performance, and Cost Parameters for a Typical Network System—L = 800 m, V_{av} = 10 m/s

$A^{1/2}$	$3L$ = 2.4 km		$5L$ = 4 km	
Guideway	*Two-Way*	*One-Way*	*Two-Way*	*One-Way*
v (fig. 4-18)	1.21	1.44	0.99	1.21
$<L_t>$, eq. (5.7.2)	2.90 km	3.46 km	3.96 km	4.84 km
$\mathcal{L}$ (eq. 4.5.1)	19.2 km		48 km	
n_s (eq. 4.5.4)	24		60	
$N/\bar{t}_d$ (eq. 5.5.2)	8.50	10.15	26.10	31.91
$T_{av}\bar{t}_d$ (eq. 4.5.18)	465 s (232 s)	195 s	379 s (189 s)	155 s
$f_{av}/\bar{t}_d$ (eq. 4.5.19)	7.7 people/hr	18.5 p/hr	9.5 p/hr	23.2 p/hr
$fs_{av}/\bar{t}_d$ (eq. 4.5.20)	2.13 p/hr	4.28 p/hr	1.92 p/hr	3.84 p/hr
$T_{max}\bar{t}_d$ (eq. 4.5.22)	970 min	485 min	2766 min	1383 min
$C_1$$	0.120	0.144	0.164	0.201
$C_2$$/day-h	12.33	9.40	13.17	9.93

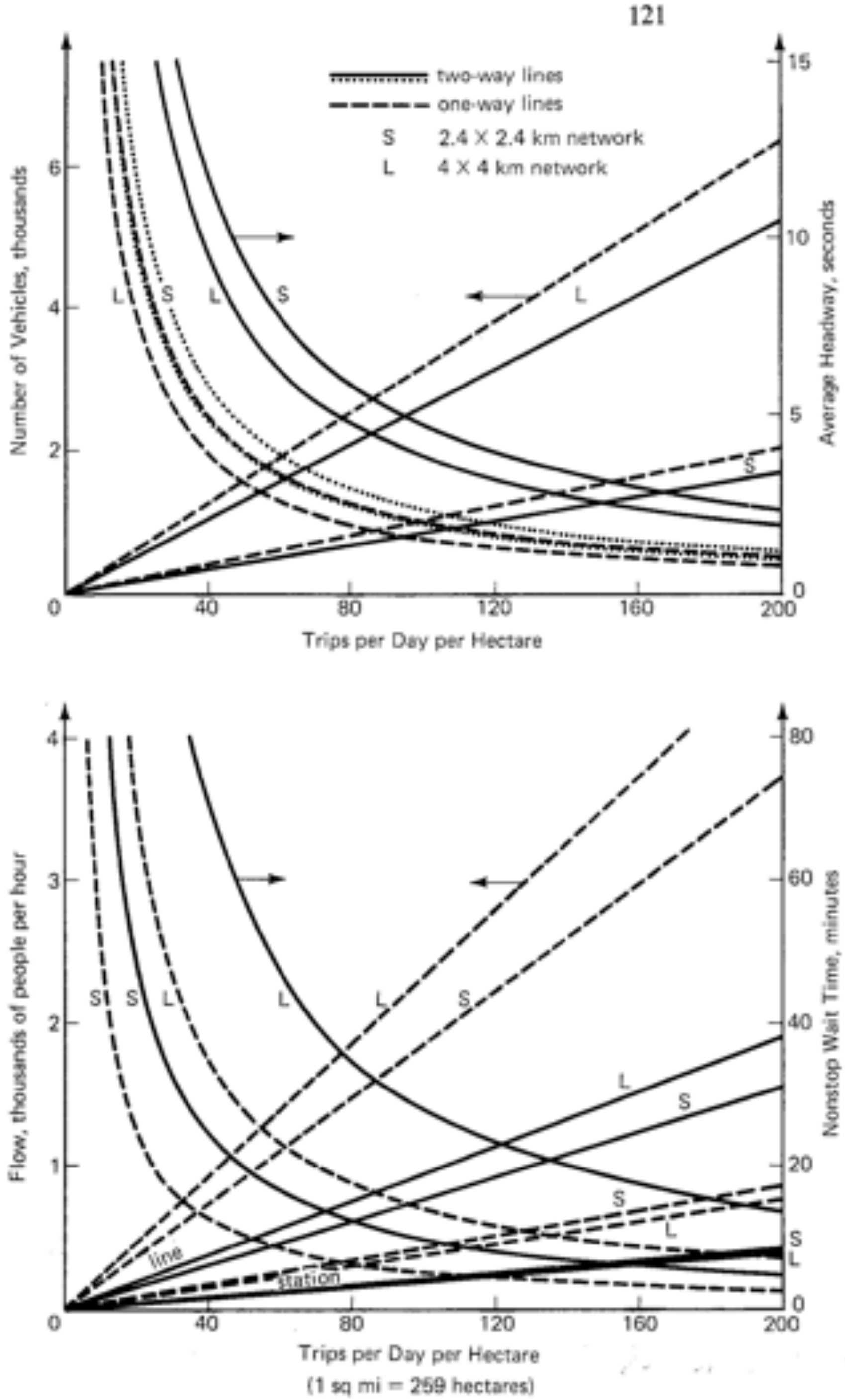

Figure 5-6. Average Performance Parameters in a Network System

of the guideway and the two groups of vehicles do not interact. If, however, the two-way network uses Y-interchanges, the average headway between merge and diverge points is not doubled. This headway is indicated by the dotted curves. It is seen in figure 5-6 that the average headway is a stronger function of provision of one-way or two-way guideways than of the size of the network. The capacity constraint on the system is due to the minimum headway, which is a fraction of average headway. The ratio T_{min}/T_{av} depends on the nonuniformity of demand, and the lines and stations should be located to make this ratio as near unity as practicable. Knowledge of T_{av} gives a feeling for the probable range of T_{min}, but T_{min} must be determined from a detailed operational simulation.

In the lower graph of figure 5-6, the lines proportional to trip density give the average line and station flow. The upper four lines marked "line," give the average line flow, and it is seen that even for the very high trip density of 200 trips per day per hectare (51,800 trips per day per square mile) the average flow is under 2000 persons per hour for two-way lines, but in the range of 4000 persons per hour for one-way lines. The maximum flows exceed these values by the ratio T_{av}/T_{min}, as discussed above. The average station flows can be compared with published data[6] from simulations on the maximum flows obtainable. With single-platform stations, flows of 600 to 1000 vehicles per hour are achievable according to the simulations.

The nonstop wait time, computed from equation (4.5.22) and presented as the family of hyperbolas in figure 5-6, is important from the viewpoint of the type of service provided. The reader is referred to the discussions following equation (4.5.22) for an interpretation of the meaning of T_{nswt}. Since the average trip time $<L_t>/V_{av}$ ranges, from table 5-1, between 4.83 min and 8.07 min, it is seen from figure 5-6 that the nonstop wait time is equal to or less than the average trip time only for densities above about 180 trips per day per hectare. The implication is that a service concept in which the first rider to board a vehicle must wait, say, at least T_{nswt} to see if another party can board going to the same stop will more than double the fleet size needed if the vehicle leaves when the first party boards. Such service will also substantially decrease patronage because the total trip time is more than doubled. Thus, group riding services require many intermediate stops, which also increase the total trip time and hence the cost of the vehicle fleet. Group services may in some cases be of interest in handling particularly high patronage between a pair or a small number of points if the headway requirements cannot be satisfied with single-party service; however, in these cases it should be determined if it would reduce the cost per trip by splitting the line into a pair of single party service lines.

Figure 5-7 shows the total cost per trip of the Cabintaxi system as a function of trip density. By comparing with figure 5-2 for given parameters, one can see under what circumstances the automated system has a lower cost per trip than a bus system, and it is seen that the comparison is favorable to the automated system for the higher range of trip densities, above about forty trips per day per hectare (10,400 trips/mi^2). It is cautioned that this comparison should not be taken too literally because of sensitivity to parameter changes and that specific conclusions should only be drawn from more detailed analysis of specific cases. In figure 5-7, it is seen that the two-way system is more expensive per trip for the large network below about 85 trips per day per hectare, and for the small network below about 130 trips. The two-way system is cheaper at high trip density because fewer vehicles are required and the vehicle cost term becomes more dominant as trip density increases. The larger network has higher cost per trip because the average trips are longer. Note that below 40 trips per day per hectare the estimated costs are very sensitive to errors in estimation of patronage.

At the bottom of figure 5-7, the modal split to the transit system is plotted as a function of trip density in accordance with equation (5.3.12). In this equation, the term ρ is to be interpreted not as the residential population density but as the number of people per hectare who live, or work, or shop, or seek recreation within the area of the transit network. If the network is placed in an area of major activity within the urban area, the latter density exceeds the residential population density by a large factor; however, if the network covers an entire city, the two average densities are roughly the same. Thus, it can be appreciated that, as the network grows, the cost per trip must increase if the modal split remains constant. However, a larger network puts more destinations within reach and can therefore be expected to increase the modal split, thus reducing the cost per trip. The plot of mode split versus trip density is made for the specific case of a mobility of three trips per person per day. This is representative of cities like Denver and Minneapolis, but, in cases in which a different value is more appropriate, the plot can be adjusted accordingly. The mode split in figure 5-7 includes trips totally within the network area as well as trips part within and part without. These mode splits will of course generally differ, and the differences must be taken into account in more detailed analysis.

A cost per trip in the range of thirty cents requires a trip density of eighty trips per day per hectare. With $\tau_d = 3$ and $m_t = 30$ percent, $C/\text{tr} = 30¢$ requires $\rho = 89$ people per hectare or 23,000 people per square mile. This is a low density for an active central business district, but $m_t = 30$ percent is well above that obtained by conventional distribution systems. Thus, to make the guideway system feasible, some auto-restrictive policies in the network area may be needed. If the network is used for freight movement, the cost per passenger trip may be reduced up to about 25 percent, as indicated in the discussion of equation (5.2.7).

The cost per passenger-kilometer, as defined by equation (5.2.4), is

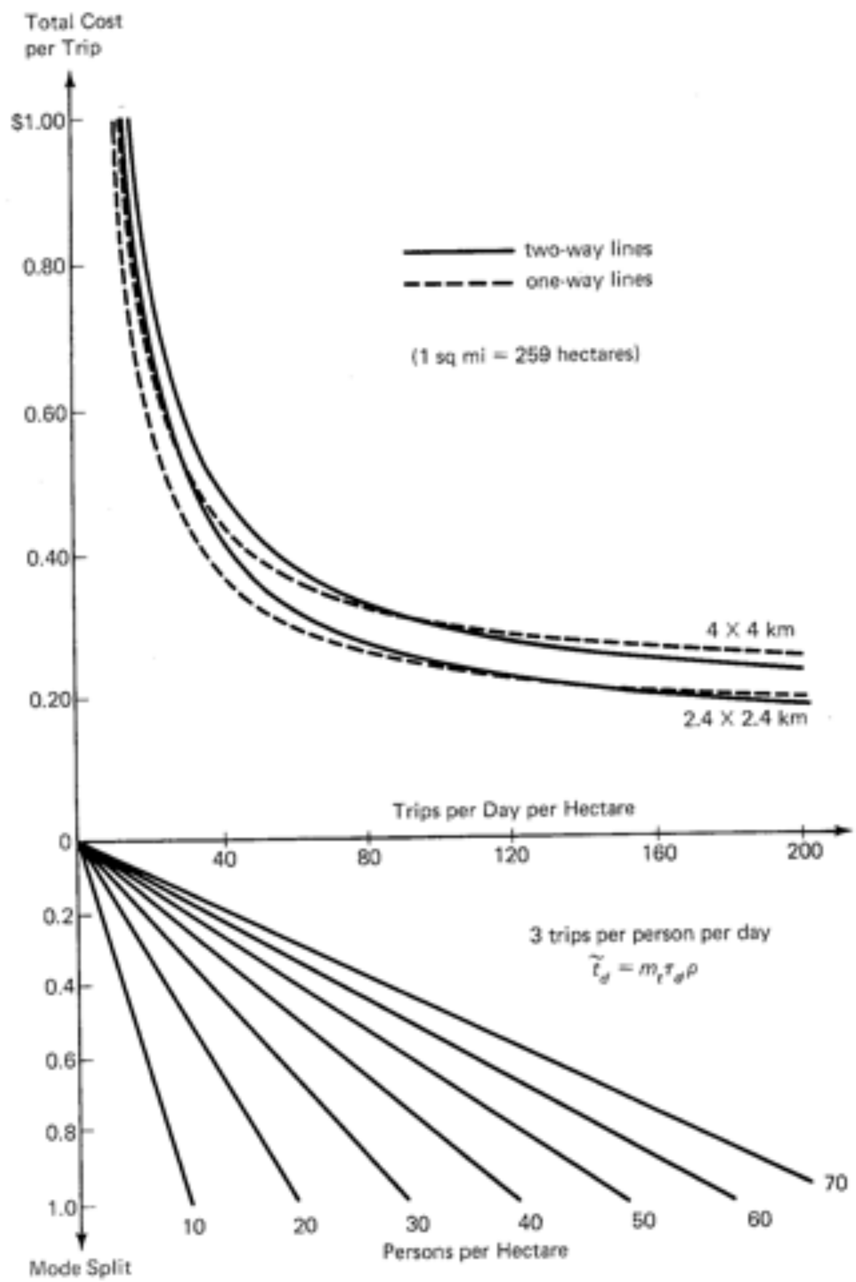

Figure 5-7. The Cost per Trip in a Network System

plotted in figure 5-8, based on the data of table 5-1. Note that there is a small economy of scale in this parameter, and that the one-way system is somewhat lower in cost per passenger-kilometer for all trip densities; whereas, if the comparison of costs is based on the trip, the one-way system is cheaper at low trip density but more expensive at high trip density. Again, at low trip density, the economic analysis is extremely sensitive to errors in estimation of patronage.

For network systems, it is worthwhile to consider the cost effectiveness parameter, PV_N, given by equation (5.2.13). Let the trend system in equation (5.2.10) be the auto system and assume the auto cost per vehicle-kilometer is in the range of 9¢ to 15¢ (15¢ to 25¢ per vehicle mile). Assume also that the average trip length is the same by both modes. Then

$$(C/\text{yr})_{\text{trend system}} = 300 t_d \frac{<L_t>}{p_v} (C/\text{veh-km})_{\text{auto}} \qquad (5.7.5)$$

in which $t_d = \tau_d \rho A'$ is the total number of trips per day. Assume the new system is part auto and part automated network, and that the mode split to the automated system is m_t. Then

$$\begin{aligned}(C/\text{yr})_{\text{new system}} = 300 t_d <L_t> \ [&(C/\text{veh-km})_{\text{auto}} \ (1 - m_t)/p_v \\ &+ (C/\text{pass- km})_{\text{net}} m_t]\end{aligned} \qquad (5.7.6)$$

Substituting equations (5.7.5) and (5.7.6) into equation (5.2.10),

$$(CS/\text{yr})_n^\circ = 300 \ \bar{t}_d A' <L_t> \left[\frac{1}{p_v} \ (C/\text{veh-km})_{\text{auto}} - (C/\text{pass-km})_{\text{net}} \right] \qquad (5.7.7)$$

in which $\bar{t}_d = m_t \tau_d \rho$.

In this simple example, assume the system is all built at once and then that $(CS/\text{yr})_n^\circ$ is the same each year. The sum in equation (5.2.13) can then be written in the following closed form:

$$PV_N = \frac{(CS/\text{yr})_n^\circ}{d - i} \left[1 - \frac{1}{(1 + d - i)^N} \right] \qquad (5.7.8)$$

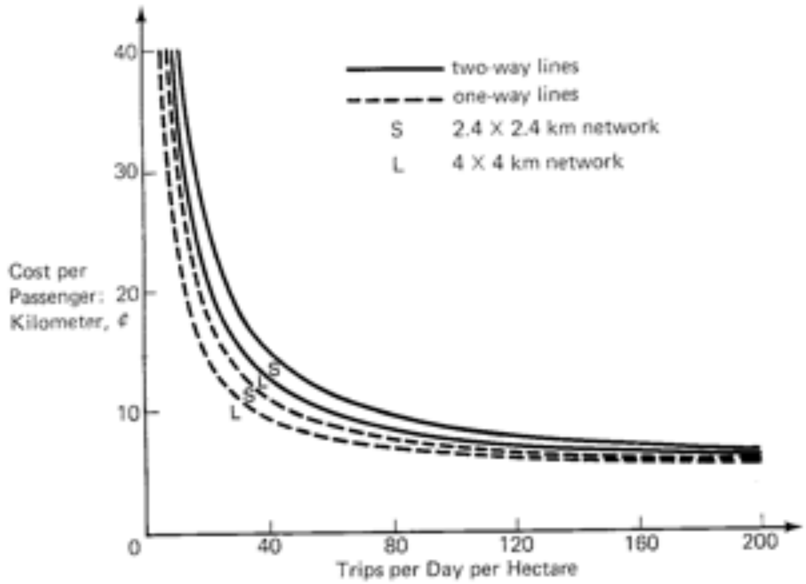

Figure 5-8. The Cost per Passenger Kilometer

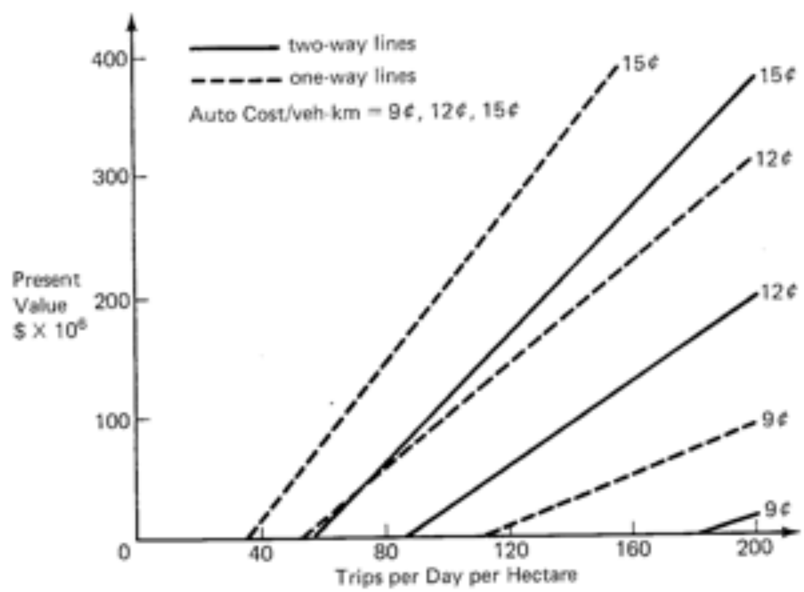

Figure 5-9. The Present Value of Future Savings If Network Is Built

For illustrative purposes, consider the specific example in which the discount rate d exceeds the inflation rate i by 2 percent, and $N = 20$ years. Then equation (5.7.8) becomes

$$PV_N = 16.4(CS/\text{yr})_n^o \tag{5.7.9}$$

Economists do not agree on the most appropriate value for $d - i$, therefore a range of values must be used and the results compared. In figure 5-9, the present value of future savings over a period of 20 years is plotted for the 4 × 4 kilometer network of figure 4-17 for $p_v = 1.5$, for a range of auto costs, and for the range of trip densities for which PV_N is positive. It is interesting to note by comparing with figure 5-8 that the present value is negative in the range below 40 trips per day per hectare in which the cost curves rise steeply. It is also noted, from figure 5-8, that the present value would be less for the smaller network. A trip density of 40 trips per day per hectare corresponds at $m_t = 0.30$ to a density of 45 persons per hectare or 11,500 persons per square mile, or at $m_t = 0.50$ to a density of 27 persons per hectare or 6900 persons per square mile. Thus, for a wide range of existing urban densities, the automated system looks attractive from the standpoint of direct cost savings if mode splits in the indicated range are achievable. To achieve mode splits in this range, however, may require the imposition of policy restrictions on auto use such as high parking fees and narrowing of streets, by converting them partly or wholly into malls.

In a real situation, it would be desirable to build the network and put it into service stage by stage. Then, in the present value calculation, $(CS/\text{yr})_n^o$ changes from year to year and equation (5.2.12) must be used directly. Such a calculation is carried out in the author's paper in *Personal Rapid Transit III* for an exponential urban density model[1].

5.8 Summary

In this chapter, basic system cost equations are first derived applicable to any transit system. Then a family of cost effectiveness parameters are developed. The most fundamental of these is the total cost per trip, meaning the annualized capital cost plus annual operating and maintenance costs divided by the annual patronage. This parameter directly indicates the percentage of subsidy required for a given fare, and, if it is in a good range, the other parameters are usually satisfactory also. However, for a variety of purposes, other cost effectiveness parameters are derived. These are discussed as follows:

1. The cost per vehicle trip. This is of interest in comparing certain

systems, but is not a parameter of fundamental importance in the economics of transit systems.

2. The cost per passenger kilometer, which is of interest in comparing transit systems with the automobile, and is ~~the most~~ a fundamental economic unit of transit performance.

3. The cost per passenger trip if freight is hauled on the transit system. If freight is hauled, more vehicles and more stations are needed, thus increasing the cost, but more revenue is generated, thus reducing the cost. Equation (5.2.7) includes both of these effects and shows, by the example given, that the potential for reduction of passenger cost per trip if freight is hauled is about 35 percent.

4. The annual surplus, which is of obvious interest to transit agencies, and to legislative bodies responsible for taxes to support the system if the surplus is negative, which is usually the case.

5. The added cost per trip required to attract one additional passenger per year. This marginal-cost parameter shows the point at which further expansion of the transit system cannot be justified on a direct economic basis. If all indirect costs are included with the direct costs, it is a true indication of the point at which to stop expansion.

6. The present value of future savings if the system is built. This parameter is developed by estimating the total transportation cost per year in each future year, say twenty years in the future, in the area in which a new transit system is to be deployed or extended, and is determined first without the new system and then with it. If the difference between these quantities is positive in a particular year, there is a cost savings in that year if the new system is built. If the savings in each future year is discounted to the present time and summed, the result is an indication of the size of research and development effort that can be mounted to bring the new system into being. If the cost includes all indirect as well as direct costs, and the accumulated present value is not strongly positive, the new system cannot be justified. This is a much stronger indication of the importance of the project to society than the more commonly used benefit/cost (b/c) ratio, which is too subjective in application. To use the b/c ratio, one must make a subjective judgment as to how far above unity it should be to justify the project, and it provides no quantitative information about costs.

In use of any of the cost effectiveness indicators, it is the responsibility of the analyst to compute a range of these indicators as a function of each of the variables to give the policy maker a sound basis for decision and a knowledge of the consequences of error.

In the third part of this chapter, the cost effectiveness equations are applied to each of the four basic types of transit systems listed in table 4-5. Network bus systems are discussed first. The cost per trip is given by equation (5.3.5) in terms of the scheduled headway and the trip density. But, as the trip density increases, a point will be reached at which the bus capacity is inadequate for the given headway. To increase patronage further, the headway must be reduced or lines must be placed closer together, in either case adding more buses in proportion to the added patronage. In this case, the cost per trip is independent of patronage and is given by equation (5.3.8) in terms of the saturation value of bus occupancy, that is, the maximum number of people the bus system can handle divided by the number of buses. Thus, the saturation value is well below the saturation occupancy of a given bus. For a typical case, the cost per trip of network bus systems is plotted in figure 5-2. The horizontal lines indicate the minimum cost per trip at saturation. The steeply rising curves, away from saturation, depict the situation of contemporary bus systems. As the population density has decreased, the trip density has decreased more than in proportion because lower density means longer service intervals at a given cost, and hence greater attractiveness of the automobile. Steeply rising cost per trip curves means rapidly increasing deficits, which lead to reduced service in terms of the number of buses or increased fares, either of which reduces patronage further.

Next, the cost effectiveness of shuttles is considered. Equation (5.4.6) shows that the cost per vehicle trip of a shuttle is a quadratically increasing function of the length of the shuttle. Thus, while relatively short shuttles have found practical applications, longer ones quickly become prohibitively expensive. Figure 5-3 shows a family of typical cases.

The cost per trip of loop systems is given by equation (5.5.4). Below it the variation of cost per trip with its parameters is discussed. A point worthy of emphasis is that, based on the data of figure 5-1, the capital cost of a vehicle per unit capacity is independent of vehicle capacity. Therefore the portion of the cost per trip due to vehicle capital cost is not a function of vehicle size but only of load factor, that is, relative occupancy. In most cases, the guideway cost term dominates. It is shown in chapter 10 that in urban applications the required guideway mass per unit length is proportional to the vehicle mass per unit length. Figure 5-4 shows that the mass per unit length of transit vehicles increases rapidly with vehicle capacity, indicating that minimum guideway size and hence cost is obtained with minimum vehicle size.

In section 5.6, equation (5.5.4) is applied to the analysis of the cost per trip components for line-haul systems. Typical numerical values of the various parameters are used to give the reader a feeling for the relative importance of vehicle, guideway, and station terms. It is shown that the unit cost of guideways must be very low compared to contemporary values if the guideway term is to reduce to the neighborhood of the other two terms. It is also shown that the introduction of automation is a significant factor in reduction of system cost only if the size of the vehicles is substantially reduced from current practice.

Finally, in section 5.7, the cost per trip equation is modified for use in analysis of the cost effectiveness of network systems. The result is equation (5.7.3). As a specific example, performance and cost effectiveness curves are developed for the case of a specific network system for which cost data is available. The basic performance parameters are shown in figure 5-6, and the cost effectiveness is shown by means of figures 5-7, 5-8 and 5-9. Note in particular that the nonstop wait time is, in almost all cases, too long to make it practical to have vehicles wait for a second party if the trip is to be nonstop. The cost per trip curves of figure 5-7 should be compared with the corresponding curves for bus systems, figure 5-2. It is seen that at high trip densities, the guideway system is cheaper. Note that, with the bottom chart relating trip density to mode split, figure 5-7 shows the range of parameters for which the guideway system is an economically justifiable alternative, and that in many cases the implication is that some form of auto-restriction policy is needed if the guideway system is to be economical. Figure 5-8 shows an economy of scale in going to large networks if the trip density does not decrease too much as the network size increases, and that the one-way line system gives a significantly lower cost per trip than the two-way line system. Finally, figure 5-9 shows how high the trip density must be if construction of the system is to be justified on a direct economic basis in comparison to an automobile system. Note from figures 5-7 and 5-8 that in the region of trip density in which the system is expensive, the cost per trip is very sensitive to errors in computing patronage; however, if the trip density is above about forty trips per hectare, the system is quite economical and insensitive to errors in computing patronage.

Problems

1. Municipal bonds at 4 percent interest are used to finance a public investment of $120,000,000 which has an estimated useful life of 40 years. What is the annual cost for capital and interest?
2. A city of 200,000 people with an average population density of 8000 people per square mile desires to install a network of scheduled bus lines using 60 passenger buses. The average line spacing for the network is 0.75 mi. It is determined that the average bus speed will be 11 mi/hr and that the average load factor can be no higher over the whole city than 25 percent. The average trip length can be taken as 40 percent of the square root of the area of the city. The cost parameters are those given in the text. It is proposed that a fare of 30¢ per trip be charged. Assume the mode split is 1.2 times the fraction of the area of the city that can be reached from an arbitrary point in the city without transferring, assuming that people will walk up to 0.25 mi from a bus line.

a. Write an equation for the modal split in terms of the area of the city (see section 6.8), and compute it for the given case.
b. Compute the total number of trips per day using parameters suggested in the text, and the number of transit trips,
c. Compute the required number of buses if the transit authority choses to base the number on the average flow in the busiest hour.
d. Compute the headway in minutes needed to achieve the computed patronage.
e. Compute the cost per trip and the annual surplus per resident.
f. If the area of the city increases by 2 times and 5 times, what is the annual surplus per resident if the headway and population density remain the same?
g. If the bus speed decreases by 30 percent due to increased street congestion, how does the annual surplus per resident change from that computed in *e*?

3. It is proposed to establish a line-haul commuter service between a major urban center and a satellite city 100 km away. Discuss the economics of this proposal in terms of the amount of travel needed to make it pay, the size satellite city implied, and the guideway cost. Use cost data discussed in the text.

References

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4. A.M. Hamer, *The Selling of Rail Rapid Transit,* Lexington Books, D. C. Heath and Company, Lexington, Mass., 1976.

5. *Nutzen-Kosten-Analyse für das Cabintaxi,* Text band, Wibera Wirtschaftsberatung Aktiengesellschaft für das Bundesminister für Forschung und Technologie, 15 July 1975.

6. *Personal Rapid Transit II,* op cit., pp. 439-478.