8 Life Cycle Cost and Reliability Allocation

.1 Introduction

he life cycle cost of a system is the sum of the acquisition cost and the ıpport cost. The acquisition cost is the purchase price plus the interest ost (see Appendix A); and the support cost is the cost of labor, equipment, pare parts, and the associated logistics required to operate the system and keep it in operation during its useful life. Every chapter in this book deals irectly or indirectly with the problem of minimization of the acquisition or ıpport cost of transit systems, and it is found that the costs vary widely epending on the choice of a large number of parameters. In this chapter, ie variation of the costs with subsystem reliability is considered.

In a given transit system, defined by the types of components used and ie service provided, the acquisition cost will generally increase with the uilt-in reliability of the components and subsystems, as shown in figure 1. On the other hand, the support costs reduce as reliability increases

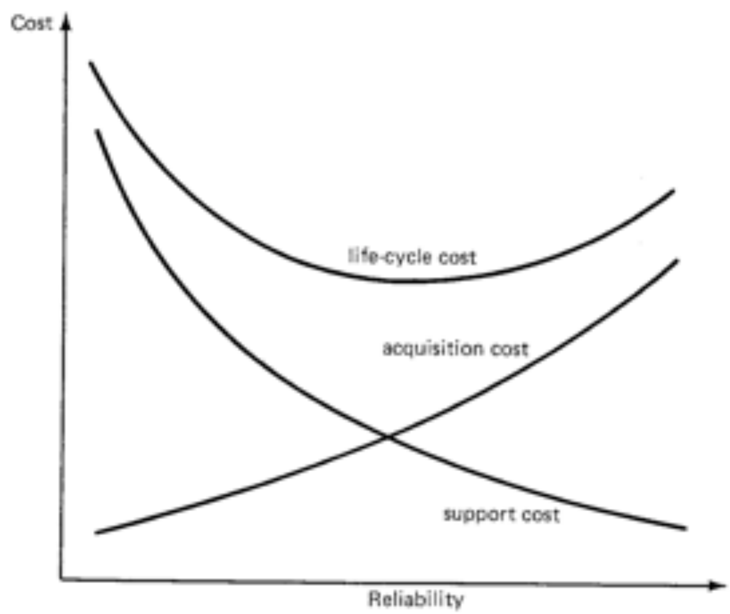

Figure 8-1. Life-Cycle Cost

because the frequency of maintenance declines. Thus the life cycle cost, as shown in figure 8-1, exhibits the character of a U-shaped curve with a single minimum point. Each subsystem, such as a motor, a controller, a braking system, or a wayside computer also possesses a similar life cycle cost curve. If each subsystem is designed so that its life cycle cost is minimum, the system life cycle cost is a minimum. If the system reliability is adequate at minimum life cycle cost, no further analysis is needed; however, the more usual situation is that in which system reliability must be increased. The problem then presents itself as to how to allocate subsystem reliabilities in such a way that the system life cycle cost is minimized at the required level of system reliability. This is a standard Lagrangian minimization problem, the solution of which is the main subject of this chapter. After completing this work, the author became aware that a similar approach had been developed by Everett[1]. The author's own original analysis of this problem has also been published.[2].

To solve the minimization problem in a meaningful way for transit systems, it is necessary to define a meaningful and accepted measure of system reliability, and to establish a means of classification of failures. System reliability is commonly measured in terms of "availability," and is treated in the next section. Classification of failures then follows.

8.2 Availability and Unavailability

Service availability in transit systems has been the subject of a great deal of analysis[3]; however, at the time of writing no completely accepted methodology has developed nor can it develop without considerably more operational experience with automated systems. Nonetheless, a logical formulation is possible which can be described in enough detail for the purpose of this chapter. The common definition of transit system availability is the ratio of the nominal trip time to the nominal trip time plus the average time delay due to failures. To take into account variations in availability in various parts of the system at various times of day and on various days, the following definition of service availability A is more suitable:

$$A = \frac{PH_{yr}}{PH_{yr} + PHD_{yr}} \tag{8.2.1}$$

in which PH_{yr} is the number of passenger-hours of travel per year on the transit system, and PHD_{yr} is the number of passenger-hours of delay due to failures per year.

Define "unavailability" as

$$\epsilon = \frac{PHD_{yr}}{PH_{yr}} \tag{8.2.2}$$

In a perfect system, ϵ vanishes. If ϵ is much less than 1, as it must be if the system operates satisfactorily, equations (8.2.1) and (8.2.2) gives

$$A = \frac{1}{1+\epsilon} \approx 1 - \epsilon \qquad (8.2.3)$$

Thus the sum of availability and unavailability is practically equal to one. Unavailability is the more useful measure of system performance because, as shown in section 8.5, it is the weighted sum of failure rates, and such a formulation is advantageous in the solution for the constrained minimum life cycle cost.

The quantity PH_{yr} can be expressed in the form

$$\begin{aligned} PH_{yr} &= \text{(Person-trips/yr)(Average trip time)} \\ &= \text{(Equivalent work days/yr)(Trips/work day)} <L_t>/V_{av} \\ &\approx 300 t_d \frac{<L_t>}{V_{av}} \qquad (8.2.4) \\ &= t_h T_{avg} T_v \qquad (8.2.12) \end{aligned}$$

in which t_d is the number of trips in an average work day, $<L_t>$ is the average trip length, and V_{av} is the average trip speed (see chapter 4). In the form given by equation (8.2.4), PH_{yr} is directly obtained from data normally available. A meaningful expression for PHD_{yr} depends upon the following definitions of subsystems and classes of failure.

8.3 Subsystems of an Automated Transit System

To make the analysis specific and therefore more meaningful, consider that an automated transit system will generally contain the types of equipment listed below:

Basic Components (without listed subsystems):

1. Vehicles
2. Guideways
3. Stations

Vehicle Subsystems:

1. Automatic vehicle door
2. Propulsion system
3. Control system including sensors and actuators
4. Power conditioning and/or supply system
5. Braking system

6. Switching system
7. Failure detection system

Wayside Subsystems:

1. Passenger processing equipment in stations (fare collection, destination selection, ticket vending, turnstiles)
2. Automatic station doors
3. Station entry monitors
4. Station-operated vehicle dispatchers
5. Merge point communication and control units
6. Diverge point communication and control units
7. Wayside switches
8. Wayside vehicle presence sensors
9. Wayside-to-vehicle, vehicle-to-wayside communication equipment
10. Central empty vehicle dispatcher
11. Central trip register and dispatcher
12. Central power supply

8.4 Classes of Failure

Each subsystem may, in general, fail in ways which produce different consequences in terms of the average number of passenger-hours of delay. These different modes of failure will be defined as different "classes of failure," and they need to be distinguished in this analysis in order to compute the number of passenger-hours of delay, and then the unavailability.

Some examples of classes of failure are the following:

Vehicle failure classes:

1. Vehicle is permitted to continue to nearest station, where passengers must egress. Vehicle is dispatched to maintenance. The number of passenger-hours of delay is the time lost by p_v passengers in transferring to second vehicle.
2. Vehicle is required to reduce speed but is permitted to continue to nearest station, where passengers must egress. Vehicle is dispatched to maintenance. The number of passenger-hours of delay is as computed in Class 1 plus time lost by people in a string of vehicles required to slow down while the failed vehicle advances to nearest station.
3. Vehicle stops or is required to stop and is pushed or towed by adjacent vehicle to nearest station. After people in the two affected vehicles egress, failed vehicle is pushed or towed to maintenance. The number of passenger-hours of delay is computed as in Class 2 but time delay is longer.

4. Vehicle stops and cannot be pushed or towed by adjacent vehicle. Must wait for rescue vehicle. The number of passenger-hours of delay is computed as in Class 3 but the total time delay is much longer and depends on the availability of alternative paths.

Merge point command and control unit failure classes:
1. Vehicles can proceed through merge point at reduced speed.
2. Vehicles must stop until unit is repaired.
3. Collision occurs.

Diverge point command and control unit failure classes:
1. Occasional vehicle is misdirected.
2. Entire stream of vehicles is misdirected.

8.5 Passenger-Hours of Delay per Year and Unavailability

Let

p = the number of different subsystems, as identified in section 8.3
q_i = the number of classes of failure of the ith type of subsystem
r_i = the number of i-type subsystem in the transit system
T_i = the number of hours the i-type subsystems are in service per year. If the subsystem is aboard a vehicle, T_i is the number of hours per year a vehicle is in service. Let this number be T_v. Typically T_v is about 10 hours/day times 300 days per year, or 3000 hours/year. If the subsystem is at wayside and the system operates 24 hours per day, $T_i = T_w = (24)(365) = 8760$ hours per year. If the system operates say six days a week and 18 hours per day, $T_w = 5616$ hours per year.
$MTBF_{ij}$ = mean time between failures of the ith class of the jth type of subsystem

The *MTBF* of interest in transit systems is that which occurs due to random failures of maintained equipment. Unlike a spacecraft, a transit system can and should undergo periodic checks at a frequency greater by a factor of at least five than the failure rates to diagnose potential failures and to replace components that wear out. The time intervals between preventive diagnostics and maintenance are therefore short compared to the *MTBF*s. In this circumstance, the probability of failure in a given time increment is not strongly a function of time and can be assumed, in the service interval, to be random. Then the number of j-class failures per year of a piece of i-type equipment is simply $T_i/MTBF_{ij}$, and the total number of failures per year is

$$\sum_{i=1}^{p} \sum_{j=1}^{q_i} \frac{r_i T_i}{MTBF_{ij}}$$

Let τ_{ij} be the mean time delay of a person involved in a j-class failure of i-type equipment, and let n_{ij} be the mean number of people involved in such a failure. Then $n_{ij}\tau_{ij}$ is the mean number of person-hours of delay due to a j-class failure of i-type equipment. Thus,

$$PHD_{yr} = \sum_{i=1}^{p} r_i T_i \sum_{j=1}^{q_i} \frac{n_{ij}\tau_{ij}}{MTBF_{ij}} \tag{8.5.1}$$

As indicated in the definition of T_i, there are generally two values for T_i, T_v for vehicle-borne equipment and T_w for wayside equipment. If there are N_v vehicles in the system, equation (8.5.1) can be written

$$PHD_{yr} = N_v T_v \sum_{i=1}^{p_{vs}} \sum_{j=1}^{q_i} \frac{n_{ij}\tau_{ij}}{MTBF_{ij}} + T_w \sum_{i=p_{vs}+1}^{p} r_i \sum_{j=1}^{q_i} \frac{n_{ij}\tau_{ij}}{MTBF_{ij}} \tag{8.5.2}$$

in which p_{vs} is the number of types of vehicle-borne subsystems, and $p_{ws} = p - p_{vs}$ is the number of wayside subsystems. The unavailability is now obtained by substituting equations (8.2.4) and (8.5.2) into equation (8.2.2)

8.6 The Constrained Minimum Life Cycle Cost

The life cycle cost of a system is the sum of the installed costs of all subsystems plus the sum of the operating and maintenance (support) cost of all subsystems. Thus it is possible to express the life cycle cost (LCC) in the form

$$LCC = N_v \sum_{i=1}^{p_{vs}} LCC_i(x_{ij}) + \sum_{i=p_{vs}+1}^{p} r_i LCC_i(x_{ij}) \tag{8.6.1}$$

in which $x_{ij} = MTBF_{ij}$ and the functional dependence of subsystem life cycle cost on reliability is explicitly indicated, that is, LCC_i is a function of the *MTBF*s for all classes of failure associated with i-type subsystems.

The problem posed is to minimize *LCC* subject to a constraint—the given value of ϵ, where ϵ is a function of all x_{ij}. To find the constrained minimum, a problem first solved by the French mathematician Lagrange (1736-1813), assume that ϵ is solved for one of the x_{ij}, say x_{mn}. Then, in principle, substitute x_{mn}, a function of all of the other x_{ij}, into *LCC*. In this case, the condition that *LCC* is minimum is

$$\frac{\partial LCC}{\partial x_{ij}} + \frac{\partial LCC}{\partial x_{mn}} \frac{\partial x_{mn}}{\partial x_{ij}} = 0 \tag{8.6.2}$$

in which i and j take all values in the ranges $j = 1, \ldots, q_i$ and $i = 1, \ldots, p$ except for the single combination of values $i = m, j = n$. Since $\epsilon = \epsilon(x_{ij})$ is a given constant,

$$\frac{\partial \epsilon}{\partial x_{ij}} + \frac{\partial \epsilon}{\partial x_{mn}} \frac{\partial x_{mn}}{\partial x_{ij}} = 0 \tag{8.6.3}$$

for all i, j except m, n.

Place the right-hand term in each of equations (8.6.2) and (8.6.3) on the right side of the equal sign and divide equation (8.6.2) by equation (8.6.3). The result can be expressed in the form

$$\frac{\dfrac{\partial LCC}{\partial x_{ij}}}{\dfrac{\partial \epsilon}{\partial x_{ij}}} = \frac{\dfrac{\partial LCC}{\partial x_{mn}}}{\dfrac{\partial \epsilon}{\partial x_{mn}}} = -\Lambda \tag{8.6.4}$$

in which, because x_{mn} could be any of the x_{ij}, Λ has the same value for all ij. The constant Λ is called a Lagrangian multiplier. $\Lambda =$

From equation (8.6.1),

$$\frac{\partial LCC}{\partial x_{ij}} = r_i \frac{\partial LCC_i}{\partial x_{ij}} \tag{a}$$

in which $r_i = N_v$ if the index corresponds to a vehicle subsystem. Similarly, from equation (8.2.2) and (8.5.1) ($x_{ij} = MTBF_{ij}$),

$$\frac{\partial \epsilon}{\partial x_{ij}} = -\frac{r_i T_i n_{ij} \tau_{ij}}{PH_{yr} x_{ij}^2} \tag{b}$$

Substituting equations (a) and (b) into equation (8.6.4), the Lagrangian multiplier becomes

$$\Lambda = \left(\frac{PH_{yr}/T_i}{n_{ij}\tau_{ij}} \right) MTBF^2_{ij} \frac{\partial LCC_i}{\partial MTBF_{ij}} \tag{8.6.5}$$

in which the substitution $x_{ij} = MTBF_{ij}$ has been made, and $T_i = T_v$ or T_w depending on the location of the equipment. The solution to the problem of the constrained minimum life cycle cost is determined by the condition that the quantity defined by the right side of equation (8.6.5) is the same for all failure classes of all subsystems.

Equation (8.6.5) contains three kinds of factors:

(1) PH_{yr}/T_i is the number of person-hours of travel on the system per hour of operation of i-type equipment, a factor determined from an understanding of the physical characteristics of the system and from an estimate of patronage.

(2) $n_{ij}\tau_{ij}$ is the number of person-hours of delay due to a j-class failure of i-type equipment. It is a matrix of values determined from classification of all failure modes, from estimation of the mean delay time due to each failure mode, and from estimation of the mean number of people involved in each failure mode. The latter factor, n_{ij}, is proportional to patronage, but since PH_{yr} is also proportional to patronage (see equation (8.2.4)), Λ is independent of patronage.

(3) The remaining factor in equation (8.6.5) depends on the reliability-cost relationship for each subsystem and is determined separately for each. The character of the function $\Lambda(MTBF)$ may be seen with the help of figure 8-1. When the slope of the life cycle cost curve is zero, $\Lambda = 0$. The solution lies to the right of this point since one would not consciously pay more for less reliability. The function $\Lambda(MTBF)$ is monotone increasing to the right of $\Lambda = 0$ if $\partial\Lambda/\partial MTBF > 0$ there. If $\Lambda(MTBF)$ is monotone increasing, it possesses a unique inverse $MTBF(\Lambda)$ and, as we will see, the problem of the constrained minimum life cycle cost has a straightforward and unique solution. To determine if $\Lambda(MTBF)$ is monotone increasing, consider the derivative of equation (8.6.5):

$$\frac{\partial\Lambda}{\partial MTBF_{ij}} = \left(\frac{PH_{yr}/T_i}{n_{ij}\tau_{ij}} \right) MTBF_{ij} \left(2\frac{\partial LCC_i}{\partial MTBF_{ij}} + MTBF_{ij}\frac{\partial^2 LCC_i}{\partial MTBF^2_{ij}} \right)$$

Thus, $\partial\Lambda/\partial MTBF_{ij} > 0$ and possesses a unique inverse if both the slope and curvature of the function $LCC_i(MTBF_{ij})$ are positive, as is shown in figure

8-1. Since it likely that LCC_i approaches infinity as $MTBF_{ij}$ approaches infinity, it is unlikely that $\partial^2 LCC_i/\partial MTBF^2_{ij}$ is ever negative, but even if it is, the curve $\Lambda(MTBF_{ij})$ is still monotone increasing if

$$\frac{\partial LCC_i}{\partial MTBF_{ij}} > \frac{MTBF_{ij}}{2} \left| \frac{\partial^2 LCC_i}{\partial MTBF^2_{ij}} \right|$$

Without more information on the functions $LCC(MTBF)$ it is not possible to prove rigorously that the above inequality always holds, but it seems highly plausible and will be assumed in the following analysis. Thus it will be assumed that $\Lambda(MTBF)$ possesses a unique inverse $MTBF(\Lambda)$ as shown in figure 8-2, but to cover contingencies, it will be assumed that if $MTBF(\Lambda)$ is not unique the lowest value is to be used. Thus, as shown in figure 8-2, if Λ is plotted as a function of $MTBF_{ij}$ for each failure class of each subsystem, the optimum value of each $MTBF_{ij}$ for the minimization of system life cycle cost can be found if the solution value of Λ for the entire system is found.

The system value of Λ is found by satisfying the given constraint on system unavailability. Combining equations (8.2.2) and (8.5.1), we can now write

$$\epsilon(\Lambda) = \frac{1}{PH_{yr}} \sum_{i=1}^{p} r_i T_i \sum_{j=1}^{q_i} \frac{n_{ij}\tau_{ij}}{MTBF_{ij}(\Lambda)} \tag{8.6.6}$$

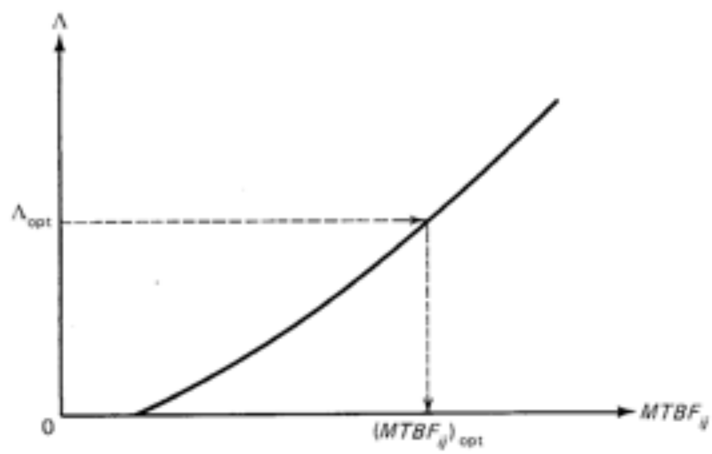

Figure 8-2. The Lagrangian Multiplier

in which the functional dependence of $MTBF_{ij}$ and hence of ϵ on Λ is indicated. Thus, the solution proceeds as follows: For each failure mode of each subsystem, $\Lambda(MTBF_{ij})$ is found and plotted. The inverse functions $MTBF_{ij}(\Lambda)$ are found from curves such as figure 8-2 and are used to compute the system curve $\epsilon(\Lambda)$ from equation (8.6.6). As indicated in figure 8-3, ϵ is maximum at $\Lambda = 0$ in the domain $\Lambda \geq 0$ and is monotone decreasing as Λ increases. The latter conclusion is a direct result of the facts (1) that all $MTBF_{ij}$ increases as Λ increases (see figure 8-2) and (2) that $\epsilon(\Lambda)$ is a sum of reciprocals of the $MTBF_{ij}$ (see equation (8.6.6)).

If $\epsilon_{spec} \geq \epsilon(0)$, where ϵ_{spec} is the specified level of system unavailability, $\Lambda = 0$ and the solution is obtained by setting all $MTBF_{ij}$ such that all $\partial LCC_i/\partial MTBF_{ij} = 0$. In the usual case, however, $\epsilon_{spec} < \epsilon(0)$. Then, as indicated in figure 8-3, the specified value of system unavailability yields a unique value $\Lambda = \Lambda_{opt}$. By entering the family of curves of Λ versus $MTBF_{ij}$ with Λ_{opt}, a unique set of values of $(MTBF_{ij})_{opt}$ are found. These values minimize system life cycle cost subject to the specified level of system unavailability.

If a given subsystem has only one class of failure there is a single set of curves like figure 8-1 for that subsystem. If in a certain subsystem there is more than one class of failure, it is implied in the above minimization process that it is possible to derive the curve $LCC_i(MTBF_{ij})$ for one particular value of j while holding the $MTBF_{ij}$ for all other j constant. It is not clear that this would always be possible, but if not, the implication would appear to be that the definition of subsystems must be further broken down.

Certainly the curves of *LCC* versus *MTBF* are not easily obtained in the early phases of a design. Preliminary reliability allocations are, however,

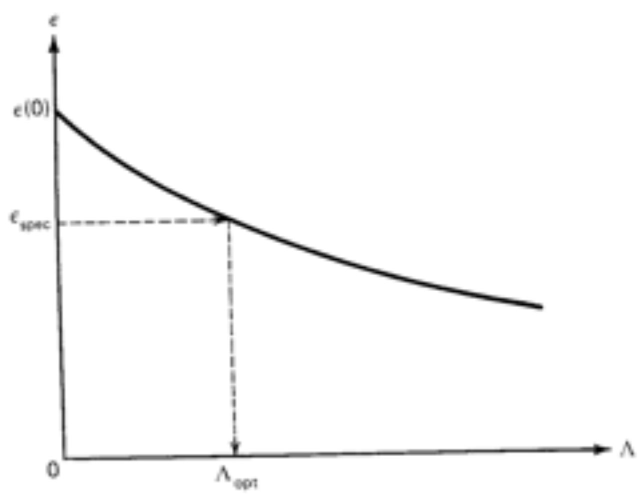

Figure 8-3. The System Constraint Function

necessary if a rational design is to ensue. Therefore, *LCC* versus *MTBF* curves must be estimated in successively more detail by a three-step process:

1. Parametric analysis of costs as functions of various system parameters
2. Refinement of costs by analogy with similar systems
3. Engineering cost analysis based on detailed designs

Out of such analysis, increasing refinement of the functions $\partial LCC_i / \partial MTBF_{ij}$ can be made, but at increasing engineering cost. As indicated in the next section, a preliminary allocation of subsystem *MTBF*s can be made without life cycle cost data; then, in section 8.8, it is shown how to obtain the next level of approximation based on preliminary values of $\partial LCC_i / \partial MTBF_{ij}$.

8.7 Approximate Solution to the Problem of Reliability Allocation

Equation (8.6.6) suggests the preliminary assumption

$$MTBF_{mn} = C n_{mn} \tau_{mn} \tag{a}$$

in which C is a constant and, to avoid confusion later, the dummy subscripts have been changed. This formula suggests that the *MTBF*s be allocated in proportion to the number of person-hours of delay due to a particular kind of failure. The constant C can be found by substituting equation (a) into equation (8.6.6). Thus

$$C = \frac{1}{\epsilon_{\text{spec}} PH_{\text{yr}}} \sum_{i=1}^{p} r_i T_i q_i \tag{b}$$

Substituting equation (b) into equation (a)

$$MTBF_{mn} = \frac{n_{mn}\tau_{mn}}{\epsilon_{\text{spec}}} \frac{N_v T_v}{PH_{\text{yr}}} \sum \tag{c}$$

in which

$$\sum = \sum_{i=1}^{p_{vg}} q_i + \frac{T_{yc}}{T_v} \frac{1}{N_v} \sum_{i=p_{vg}+1}^{p} r_i q_i \tag{8.7.1}$$

is the sum of the total number of failure classes defined for vehicle subsystems plus a weighting factor times the number of failure classes in all wayside subsystems.

In many cases n_{mn} can be expressed in the form

$$n_{mn} = \dot{n}_m \tau_{mn} \tag{8.7.2}$$

in which $\dot{n}_m$ is the mean flow of people involved in a failure of subsystem m. (Cases in which n_{mn} is not proportional to a flow are of lesser importance to system availability and, in any case, can be treated simply by substituting n_{mn} for $\dot{n}_m\tau_{mn}$.) Thus, equation (c) becomes

$$MTBF_{mn} = \frac{\dot{n}_m \tau_{mn}^2}{\epsilon_{spec}} \frac{N_v T_v}{PH_{yr}} \sum \tag{8.7.3}$$

The strong dependence of the required reliability on the time delay due to failure, τ_{mn}, is clearly evident from equation (8.7.3), thus indicating the importance of developing operational strategies in which failures can be cleared as quickly as possible. Since $\dot{n}_m$, N_v, and PH_{yr} are all proportional to patronage, the required *MTBF* is proportional to patronage, a conclusion that is intuitively reasonable. Also, equation (8.7.3) shows that, for a given patronage, if N_v increases due to use of smaller vehicles, $MTBF_{mn}$ increases unless by design changes τ_{mn} is decreased enough so that the product $N_v\tau_{mn}^2$ does not change. Thus, if τ_{mn} varies as $N_v^{-1/2}$ the reliability requirements do not worsen in small-vehicle systems.

8.8. Approximate Solution to the Problems of Minimization of Life Cycle Cost and Reliability Allocation

Equation (8.7.3) allocates the reliability requirements in proportion to the number of person-hours of delay due to each type of failure, but makes no allowance for the possibility that the life cycle costs of some subsystems may change more rapidly with *MTBF* than others. To account in as simple a way as possible for such variations, assume in equation (8.6.5) that, in the region of interest, the slopes of the curves of LCC_i versus $MTBF_{ij}$ are constant, that is, independent of $MTBF_{ij}$. Then equation (8.6.5) can be solved for $MTBF_{ij}$:

$$MTBF_{ij} = \left(\frac{n_{ij}\tau_{ij}}{LCC'_{ij}} \frac{T_i \Lambda}{PH_{yr}} \right)^{1/2} \tag{8.8.1}$$

in which

$$LCC'_{ij} = \frac{\partial LCC_i}{\partial MTBF_{ij}}$$

If equation (8.8.1) is substituted into equation (8.6.6), the result can be solved for $\Lambda^{1/2}$. Thus

$$\Lambda_{\text{opt}}^{1/2} = \frac{1}{\epsilon_{\text{spec}} PH_{yr}^{1/2}} \sum_{i=1}^{p} r_i T_i^{1/2} \sum_{j=1}^{q_i} (n_{ij}\tau_{ij} LCC'_{ij})^{1/2} \qquad (8.8.2)$$

On substituting equation (8.8.2) into equation (8.8.1) and changing the dummy indices i,j to m,n in equation (8.8.1), the $MTBF$s are seen to be allocated according to the equation

$$MTBF_{mn} = \frac{n_{mn}\tau_{mn}}{\epsilon_{\text{spec}}} \frac{N_v T_v}{PH_{yr}} \sum_{mn} \qquad (8.8.3)$$

in which $\frac{N_v T_v}{P H_{yr}} = \frac{t_h T_{trip}}{p_v} \frac{(t_d/t_h) S}{t_d\ T_{trip}} = \frac{S}{p_v}$ $S \approx 1$ for demand resp. ≈ 2 for SLT

$$\sum_{mn} = \left(\frac{T_m}{T_v} \right)^{1/2} \sum_{i=1}^{p_{vg}} \sum_{j=1}^{q_i} \left(\frac{n_{ij}\tau_{ij}}{n_{mn}\tau_{mn}} \frac{LCC'_{ij}}{LCC'_{mn}} \right)^{1/2}$$

$$+ \frac{(T_m T_w)^{1/2}}{T_v} \sum_{i=p_{vg}+1}^{p} \frac{r_i}{N_v} \sum_{j=1}^{q_i} \left(\frac{n_{ij}\tau_{ij}}{n_{mn}\tau_{mn}} \frac{LCC'_{ij}}{LCC'_{mn}} \right)^{1/2}$$

(8.8.4)

If subscript m corresponds to a vehicle subsystem, $T_m = T_v$ and the second double summation, dependent on the wayside subsystems, is weighted by the ratio $(T_w/T_v)^{1/2}$, which is greater than one if T_w is greater than T_v. If subscript m corresponds to a wayside subsystem, $T_m = T_w$ and $(T_w/T_v)^{1/2}$ factors out of equation (8.8.4). The second double sum is again weighted with respect to the first by the factor $(T_w/T_v)^{1/2}$. As indicated in section 8.5, in most cases $(T_w/T_v)^{1/2} \leqslant (8760/3000)^{1/2} = 1.7 > 1$. Thus the systems in operation longer weigh more heavily in determining the reliability requirements, as should be the case. It is also seen from equation

(8.8.4) that, since LCC'_{mn} is in the denominator, failure modes for which LCC increases more rapidly with $MTBF$ are allocated a smaller $MTBF$, the correct direction to minimize life cycle cost. Moreover, even without accounting for variations in LCC', equation (8.8.4) is more realistic than equation (8.7.1) in that failure modes for which $n_{ij}\tau_{ij}$ is larger weigh more heavily in determining the subsystem $MTBF$ requirements. Since the LCC' appear in equation (8.8.4) only under square-root signs, variations in the corresponding ratios have a diminished effect on the $MTBF$ requirements.

If all failure delay times are held constant except τ_{mn} and equation (8.7.2) can be used, $MTBF_{mn}$ is proportional to τ_{mn}, not to τ_{mn}^2 as is the case with equation (8.7.3); however, if all of the τ_{mn} are reduced in the same proportion, $MTBF_{mn}$ still reduces in proportion to τ_{mn}^2. If one of the τ_{ij} is large, all of the $MTBF_{mn}$ must suffer an increase in order to meet the specified system unavailability, ϵ_{spec}. This is clearly as it should be.

Note from equation (8.8.3) that, if equation (8.7.2) is substituted, $MTBF_{mn}$ is proportional to the ratio $\dot{n}_m/PH_{yr}$, that is, the ratio of flow rate in people per hour to person-hours of travel per year. This ratio is independent of patronage; however, N_v is proportional to patronage (see chapter 4). Therefore, the $MTBF$ requirements are proportional to patronage and to the number of vehicles in the system at a given patronage level. If the reliability requirements are not to increase in smaller vehicle systems (larger N_v), it is necessary that the operational control system be designed so that the squares of the delay times due to failures decrease in the same proportion as N_v increases, that is, that the product $N_v\tau_{mn}^2$ remain fixed. As the system size increases, PH_{yr} increases in proportion to N_v; therefore, the reliability requirements change as the system grows only insofar as the flow rates may be larger in a larger system.

Note in equation (8.8.3) that $MTBF_{mn}$ is inversely proportional to PH_{yr}. This may seem counterintuitive, but, from equation (8.2.2), at fixed ϵ an increase in PH_{yr} implies an increase in the number of person-hours of delay per year. An increase in the latter quantity clearly implies a decrease in $MTBF$ requirements.

8.9 Reliability Allocation in Sub-systems

A transit system is composed of vehicles, stations, wayside equipment, and central facilities. For the system as a whole, each of these systems is a subsystem. But a subsystem in this sense may be composed of "sub-subsystems." For example, if a vehicle is called a subsystem, its propulsion system, braking system, control system, and so forth, may be called "sub-subsystems." Each of these "sub-subsystems" can further be broken down into components or "sub-sub-subsystems." In application of the theory culiminating in equations (8.8.3, 8.8.4) the classification of subsystems is arbitrary. The user should, however, pick as "subsystems" the largest units for which specific failure consequences can be defined. Such a unit may be an entire vehicle because failures of its "subsystems" produce consequences such as defined in section 8.4 regardless of which "subsystem" failed. Similarly, several different types of station failures may cause identical consequences in terms of passenger delay. The requirement for selection of subsystems is that it be possible to derive a specific value of $n_{mn}\tau_{mn}$ for each of its classes of failure.

A "class of failure" may or may not be uniquely identified with a specific component or sub-subsystem failure. If it is, then the corresponding $MTBF_{mn}$ uniquely defines the required $MTBF$ of a specific component or subsystem. If not, any one of a number of failures can cause a failure of class "mn." In the latter case, one can write

$$MTBF^{-1}_{mn} = \sum_{k=1}^{K} MTBF^{-1}_{mnk} \tag{8.9.1}$$

in which there are K "sub-subsystems" or components, the failure of any one of which will cause a failure of class "mn". Equation (8.9.1) states simply that the failure rate $MTBF^{-1}$ of a failure class or "system" is equal to the sum of failure rates of a series of independent units, the failure of any one of which produces a "system" failure.

But the theory of equations (8.8.3, 8.8.4) defines $MTBF_{mn}$. Then equation (8.9.1) may be considered as a constraint equation upon the basis of which the sub-subsystem $MTBF$s can be allocated to minimize the life cycle cost of the subsystem. Thus, replace ϵ in equation (8.6.4) by $MTBF^{-1}$. Then

$$\frac{\partial MTBF^{-1}}{\partial MTBF_k} = -\frac{1}{MTBF_k^2} \tag{a}$$

Equation (8.6.4) now takes the form

$$\Lambda = MTBF_k^2 \frac{\partial LCC_k}{\partial MTBF_k} \tag{b}$$

Following the derivation of equations (8.8.3, 8.8.4), solve equation (b) for $MTBF_k$. Thus

$$MTBF_k = \Lambda^{1/2}(LCC_k')^{-1/2} \tag{c}$$

Substitute equation (c) into equation (8.9.1), solve for $\Lambda^{1/2}$ and drop the subscripts *mn* for brevity. Then

$$\Lambda^{1/2} = MTBF \sum_{k=1}^{K} (LCC'_k)^{1/2} \tag{d}$$

in analogy with equation (8.8.2). Now change the dummy subscript in equation (c) and substitute equation (d) into equation (c) to obtain the desired result:

$$MTBF_\ell = MTBF \sum_{k=1}^{K} \left(\frac{LCC'_k}{LCC'_\ell}\right)^{1/2} \tag{8.9.2}$$

Equation (8.9.2) shows that the *MTBF*s of each of a set of K sub-subsystems should be allocated in proportion to the known required mean time to failure of the subsystem, and weighted in inverse proportion to the square root of the corresponding slope of the sub-subsystem life cycle cost curve. If all of the LCC'_k are the same, then $MTBF_\ell = K(MTBF)$ as is to be expected, that is, if each of K components can fail in such a way as to produce a failure of the sub-subsystem of which they are a part, the failure rate of the sub-subsystem is greater than the failure rate of each of the components by the factor K.

Equation (8.9.2) together with equations (8.8.3, 8.8.4) lay the foundation for allocation of reliability requirements of all components and subsystems in a system of any degree of complexity.

8.10 Simultaneous Failures

The form of equation (8.5.1) assumes that failures act independently, that is, that if two failures were to occur simultaneously the total number of person-hours of delay would simply be the sum of the corresponding terms for independent failures. This is clearly not always the case because it is possible that the simultaneous occurrence of two independent failures could cause a collision. If precautions have not been taken in advance to minimize the consequences of collisions, the sum of the $n_{ij}\tau_{ij}$ for two simultaneously acting failure modes could greatly exceed the corresponding sum if the two failures occur at different times.

Strictly speaking, then, we should add to equation (8.5.1) terms corresponding to interactive failures. These terms will contain products of the $MTBF_{ij}$ in the denominators and, in the differentiation process leading to equation (8.6.5) and the subsequent equations for required *MTBF*s, will lead to fundamental complications—$\partial\epsilon/\partial x_{ij}$ becomes a function of all interactive failure modes, not just of x_{ij}. But, in a well-designed system, the probability of collisions involving greatly increased delay must be very small. Therefore, it is better to use the theory developed and to proceed iteratively to consider the consequences of simultaneous failures. The following procedure is recommended: First compute the required *MTBF*s from equation (8.8.3). Then, having the required *MTBF*s for individual failures, compute the *MTBF*s for simultaneous, interactive failures and estimate the corresponding $n_{ij}\tau_{ij}$ for them. If the corresponding contributions to equation (8.6.6) add significantly, to ϵ, then a new smaller ϵ_{spec} must be defined and the calculation repeated until the ϵ_{spec} plus the ϵ corresponding to collisions does not exceed the desired ϵ_{spec} counting all failures. A case of simultaneous failures is considered in section 9.5.

8.11 Summary

A method is developed for allocation of the reliability requirements of the subsystems and sub-systems of an automated transit system in such a way that life cycle cost is minimized. Besides a complete classification of the subsystems and their failure modes, the method requires knowledge of (1) the yearly number of hours of operation of the vehicle-borne and wayside equipment, (2) the mean number of person-hours of delay due to each failure (failure effects analysis), and (3) the slopes of the curves of subsystem and sub-subsystem life cycle cost versus *MTBF*.

The solution is given by equations (8.6.5) and (8.6.6); however, using it the numerical solution is graphical. An analytic approximation, adequate if the variation in the slopes of the life cycle cost curve are small, is given by equations (8.8.3, 8.8.4) and equation (8.9.2). The latter solutions have the additional advantage of providing a great deal of insight into the behavior of *MTBF* requirements with various parameters, for example, the *MTBF* requirements are:

1. Proportional to patronage
2. Independent of system size
3. Proportional to the square of the time delays due to failure
4. Proportional to the number of vehicles.
   Thus, if, with a given patronage, the vehicle size is reduced so that N_v increases, $\tau^2_{\min}$ must be caused to decrease in the same proportion if the *MTBF* requirements are not to worsen. Thus, more sophisticated control systems are required in small-vehicle systems than in large-vehicle systems.

References

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