Transit Systems Theory

J. Edward Anderson
University of Minnesota

Lexington Books
D.C. Heath and Company
Lexington, Massachusetts
Toronto

Library of Congress Cataloging in Publication Data

Anderson, John Edward.
Transit systems theory.

Includes bibliographical references and index.
1. Urban transportation. 2. Local transit.
I. Title.
TA1205.A5 388.4 77-011856
ISBN 0-669-01902-X

Copyright © 1978 by D.C. Heath and Company

All rights reserved. No part of this publication may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopy, recording, or any information storage or retrieval system, without permission in writing from the publisher.

Published simultaneously in Canada

Printed in the United States of America

International Standard Book Number: 0-669-01902-x

Library of Congress Catalog Card Number: 77-011856

DC Heath and Company
Lexington Books
125 Spring Street
Lexington MA 02173

617 862 6650

DC Heath
A Raytheon Company

COPYRIGHT ASSIGNMENT

For good and valuable consideration, the undersigned D. C. HEATH and COMPANY with a place of business at Lexington, Massachusetts, herein called the Assignor,

HEREBY SELLS, ASSIGNS, TRANSFERS and SETS OVER unto J. Edward Anderson located at Boston University, Boston, Massachusetts, herein called the Assignee, all the Assignor's right, title and interest in and to (a) the copyright secured on a book entitled TRANSIT SYSTEMS THEORY by Assignee, which was published in the United States of America on May 31, 1978, and was duly registered for copyright in the United States Copyright Office in the Assignor's name under date of June 6, 1978, Reg. No. TX 51-861; and (b) any and all renewals and extensions of such copyright that may be secured under the laws now or hereafter pertaining thereto in the United States,

TO HAVE AND TO HOLD the same unto the Assignee, its successors and assigns, for and during the existence of the copyright, and all renewals and extensions thereof, IN WITNESS WHEREOF the Assignor has caused this assignment to be signed in its corporate name by its duly authorized officer this 10th day of December, 1988.

D. C. HEATH AND COMPANY

By

Robert Bovenschulte
Title Vice President

Comments on *Transit Systems Theory*, 1978

With scanning technology now easily available and the Copyright assigned back to me, I have scanned my textbook *Transit Systems Theory* into my computer and offer it hereby via the Internet to any interested person. While I have often thought about developing a new edition, such a task is a major project that would likely take at least two years, and which I have not set aside other work to undertake. I have preferred to continue to work to commercialize systems. People who know me know, however, that I have published many papers on topics related to transit systems theory since the release of this book, and particularly on the new automated small-vehicle network systems often called PRT. Many of these papers can be found on www.prtnz.com, http://kinetic.seattle.wa.us, and http://gettherefast.org.

I have here only a few comments on my 1978 book.

The material in Chapter 3 is expanded into three dimensions in my report "Maglev Performance Simulator," Report of Contract No. DTRS-57-94-C-00004, U. S. Dept. of Transportation, February 19, 1994

The material in Section 4.3 “Loop Systems” has been completely superseded by my paper “Calculation of Performance and Fleet Size in Transit Systems,” *Journal of Advanced Transportation (JAT)*, 16:3(1982)231-252.

The material in Section 4.5 “Network Systems” is augmented by my paper "Personal Rapid Transit: Matching Capacity to Demand," an Advanced Transit Association paper, February 1998.

The material in Chapter 5 is augmented fundamentally by the work in my paper "Optimization of Transit-System Characteristics," *JAT*, 18:1(1984):77-111.

The material in Chapter 7 is augmented by the work in my paper "Safe Design of Personal Rapid Transit Systems," *JAT*, 28:1(1994):1-15.

The work of Chapter 8 is augmented and expanded by the work in my papers "Dependability as a Measure of On-Time Performance of Personal Rapid Transit Systems," *JAT*, 26:3(1992): 101-212, and "Life-Cycle Costs and Reliability Allocation in Automated Transit," *High Speed Ground Transportation*, 11:1(1977):1-18.

The work of Chapter 9 is augmented and expanded by the work in my paper “Failure Modes and Effects,” www.prtnz.com

The first two sections of Chapter 10 “Guideway Structures” were influenced as I wrote by my work on the Cabintaxi PRT system, which used a box-beam guideway. Subsequently I have extended that work to a U-shaped guideway similar to that proposed by The Aerospace Corporation.[1] The analysis given in Chapter 10 of dynamic loading is not dependent on the cross section.

The work of Chapter 11”Design for Maximum Cost Effectiveness” is augmented and extended by a number of my papers, for example

> "Automated Transit Vehicle Size Considerations," *JAT*, 20:2(1986):97-105.
> "What Determines Transit Energy Use," *JAT*, 22:2(1988):108-132.
> “A Review of the State of the Art of Personal Rapid Transit.” *JAT*, 34:1(2000).
> “The Future of High-Capacity PRT”, Advanced Transit Association Conference, Bologna, Italy, 2005.

The engineering science of control of automated guideway transit systems was during the 1970s the subject of the work of many engineers.[2] While I had worked for a number of years at the Honeywell Aero Research Department on the control of military aircraft and spacecraft, I decided to devote my attention to a problem that had received too little attention: The characteristics of the system that deserved to be controlled. That is the subject of my 1978 *Transit Systems Theory*. Subsequently, however, as a result of having to develop beginning in 1981 after thirteen years in the field all of the components of a new PRT system I developed the necessary control system and report some of my work on control in the following papers:

> "Synchronous or Clear-Path Control in Personal Rapid Transit," *JAT*, 30:3(1996):1-3.
> "Longitudinal Control of a Vehicle," *JAT*, 31:3(1997):237-247.
> "Control of Personal Rapid Transit Systems," *JAT*, 32:1(1998).
> “Simulation of the Operation of Personal Rapid Transit Systems.” *Computers in Railways VI*, WIT Press, Boston, Southampton, 1998, 523-532.

In the scanned copy contained herein I have corrected typos I have found. If the reader should find more, I would appreciate very much being informed. I can easily correct the text page by page.

J. Ed Anderson
Fridley, Minnesota USA
September 26, 2007

---

[1] Irving, J. H., Bernstein, H., Olson, C. L., and Buyan, J. *Fundamentals of Personal Rapid Transit,* Lexington Books, D. C. Heath and Company, Lexington, MA, 1978.

[2] See for example the proceedings of the 1971, 1973 and 1975 International Conferences on Personal Rapid Transit.

Contents

List of Figures xi

List of Tables xv

Foreword xvii

Preface xxi

Chapter 1 **Introduction** 1

Chapter 2 **Basic Performance Relationships** 7

2.1 The Acceleration Profile 7
2.2 The Velocity Profile and Stopping Distance 9
2.3 Acceleration Power – Const Power [illegible] 11
2.4 Trip Time and Average Velocity 13
2.5 Time and Distance Loss due to Speed Reduction 17
2.6 Average Power Consumption 18
2.7 Summary 20
Problems 21

Chapter 3 **Transitions from Straight to Curved Guideways** 23

3.1 The Differential Equations for the Transition Curve 23
3.2 The Constant Speed Spiral 26
3.3 A Right-Angle Curve at Constant Speed 28
3.4 Transition to an Off-Line Station at Constant Speed 31
3.5 The Constant Deceleration Spiral 33
3.6 The Lateral Response of a Vehicle due to a Sudden Change in the Curvature of the Path 36
3.7 Superelevation 42
3.8 Summary 44
Problems 45

leave out.
& concentrate
on rest. Too
Course is
long

Chapter 4 **Performance Relationships for Specific Systems** 47

4.1 Shuttle Systems 47
4.2 Station Throughput 53

4.3 Loop Systems 60
4.4 Line-Haul Systems 76
4.5 Network Systems 78
4.6 Summary 89
Problems 92

Chapter 5 **Cost Effectiveness** 97

5.1 Cost Equations 97
5.2 Equations for Cost Effectiveness 99
5.3 Cost Effectiveness of Bus Systems 109
5.4 Cost Effectiveness of Shuttles 107
5.5 Cost Effectiveness of Loop Systems 109
5.6 Cost Effectiveness of Line-Haul Systems 113
5.7 Cost Effectiveness of Guideway Network Systems 117
5.8 Summary 127
Problems 130

Chapter 6 **Patronage Analysis** 133

6.1 Relationship between Yearly, Daily, and Peak-Hour Patronage 134
6.2 Mobility 135
6.3 Required Precision of Patronage Estimates 136
6.4 Trip Generation 137
6.5 Trip Distribution 140
6.6 Mode Split Analysis—A Probability Argument 141
6.7 Mode Split Analysis—The Logit Model 142
6.8 Factors That Influence Patronage 147
6.9 Summary 153

Chapter 7 **Requirements for Safe Operation** 157

7.1 Introduction 157
7.2 Requirements for Collision Avoidance 159
7.3 Constant Force, Constant Displacement Shock Absorber 179
7.4 Criteria for Avoidance of Passenger Injury in Collisions 183
7.5 Collision with a Constraint Device in a Decelerating Vehicle 189
7.6 Safe Velocities of Collision between Vehicles 191

7.7 Oblique Collisions 197
7.8 Summary 198

Chapter 8 **Life Cycle Cost and the Theory of Reliability Allocation** 203

8.1 Introduction 203
8.2 Availability and Unavailability 204
8.3 Subsystems of an Automated Transit System 205
8.4 Classes of Failure 206
8.5 Passenger-Hours of Delay per Year and Unavailability 207
8.6 The Constrained Minimum Life Cycle Cost 208
8.7 Approximate Solution to the Problem of Reliability Allocation 213
8.8 Approximate Solution to the Problem of Minimization of Life Cycle Cost and Reliability Allocation 214
8.9 Reliability Allocation in Sub-systems 216
8.10 Simultaneous Failures 218
8.11 Summary 219

Chapter 9 **Redundancy, Failure Modes and Effects, and Reliability Allocation** 221

9.1 Introduction 221
9.2 Redundancy 222
9.3 Subsystems and Classes of Failure 229
9.4 Vehicle Failures 230
9.5 Station Entry Monitoring Equipment 237
9.6 Failures of Passenger-Processing Equipment in Stations 241
9.7 Merge-Equipment Failures 245
9.8 Diverge Equipment Failures 245
9.9 Failures in Wayside Communications Equipment 246
9.10 Failures in Central Control Equipment 246
9.11 Escape Mechanisms 248
9.12 Reliability Allocation 249
9.13 Summary 254

Chapter 10 **Guideway Structures** 257

10.1 Introduction 257

10.2 Optimum Cross Section Based on Bending Stress 258
10.3 Dynamic Loading—Single Vehicle Crossing a Span 268
10.4 Dynamic Loading—Cascade of Vehicles Crossing a Span 285
10.5 Limit Valve of Speed Based on Ride Comfort 292
10.6 Torsion 295
10.7 Plate Buckling 301
10.8 Plate Vibration 302
10.9 Optimum Span Length 304
10.10 Summary 308

Chapter 11 **Design for Maximum Cost Effectiveness** 315

11.1 Introduction 315
11.2 Guideways 317
11.3 Vehicle Fleet Costs 318
11.4 Propulsion and Braking 320
11.5 Standing Versus Seated Passengers 322
11.6 Reliability 323
11.7 Dual Mode Versus Captive Vehicles 324
11.8 Guideway Configurations 327
11.9 Control 328
11.10 Energy Conservation 329
11.11 Capacity Requirements 330

Appendix A **Derivation of the Amortization Factor** 333

Index 337

About the Author 341

List of Figures

2-1 The Acceleration Profile 7
2-2 The Velocity Profile 9
2-3 The Station-to-Station Velocity Profile 14
2-4 The Average Velocity 16
2-5 The Velocity Profile in Speed Reduction 17
3-1 Notation in a Transition Curve 24
3-2 A Spiral Transition to a Parallel Line at Constant Speed 31
3-3 A Sudden Transition from a Straight to a Curved Guideway 37
3-4 Lateral Damping Functions 41
3-5 A Superelevated Guideway 43
4-1 A Simple Shuttle 47
4-2 Characteristic Times for and Capacity of Simple Shuttles 49
4-3 Simple Shuttles with Intermediate Stations 50
4-4 A Two-Vehicle Shuttle 51
4-5 Motions of a Four-Vehicle Shuttle 52
4-6 The Distance-Time Diagrams of Two Successive Vehicles Entering and Leaving a Station 54
4-7 The Relationship between Minimum Permissible Headway through Stations and Vehicle Length 58
4-8 The Position-Time Diagram for an End-of-Line Station 59
4-9 Schematic Diagram of a One-Way Transit System 61
4-10 Example Computation of the Empty Vehicle Fleet 73
4-11 Line-Haul Configurations 77
4-12 An Idealized Transit Network 81

4-13 Idealization of the Average Trip Length 82
4-14 Four-Station Square Loop 83
4-15 A Two-Loop Network 83
4-16 A Five-Loop Network 84
4-17 A Thirteen-Loop Network 85
4-18 The Average Trip Length in Finite Networks with Uniform Demand 87
5-1 Guideway Transit Vehicle Cost per Unit Capacity 98
5-2 Total Cost per Trip of Bus Systems 106
5-3 The Cost per Vehicle Trip of a Typical Shuttle 109
5-4 Transit Vehicle Mass per Unit Length 112
5-5 Network Interchanges 117
5-6 Average Performance Parameters in a Network System 121
5-7 The Cost per Trip in a Network System 124
5-8 The Cost per Passenger Kilometer 126
5-9 The Present Value of Future Savings If Network Is Built 126
6-1 The Population Density Required to Achieve $\bar{t}_d$ = 40 Trips per Day per Hectare 136
6-2 Access Mode Split Functions 139
6-3 Transit Mode Split as a Function of Station Density 139
7-1 Velocity-Time Diagram for Determining No-Collision Headway in Sudden-Deceleration Failures 160
7-2 Velocity-Time Diagram for Determining No-Collision Headway in Sudden-Acceleration Failures 167
7-3 Velocity-Time Diagram for Determining Normal Stopped-Vehicle Separation in a Station 171
7-4 A Hydraulic Shock Absorber 176

7-5	Velocity-Time Diagram of a Collision	177
7-6	The Shape of a Variable Diameter Orfice Rod Varied in a Manner to Produce Constant Force in a Hydraulic Shock Absorber	181
7-7	Idealized Force-Deflection Curve for a Constraint Device	185
7-8	Collision of an Object with a Constraint Device	186
7-9	Velocity-Time Diagram of a Vehicle and an Unconstrained Passenger during Collision	192
7-10	Geometry of an Oblique Collision	197
8-1	Life Cycle Cost	203
8-2	The Lagrangian Multiplier	211
8-3	The System Constraint Function	212
9-1	Distance-Time Diagram Used to Compute the Time Delay due to Slowdown of a Vehicle	231
9-2	Position-Time Diagrams for Pushable Failure	234
9-3	Position-Time Diagrams for Passengers Waiting for Service as a Result of a Delay of Duration τ_1	243
9-4	Service Vehicle in Operation on the Side of the Guideway of the Cabinlift System	250
10-1	Cross Section of a Rectangular Beam	258
10-2	Required Cross-Sectional Area of a Box Beam at Given Maximum Static Load	261
10-3	Required Wall Thickness of an Optimum Cross-Section Steel Box Beam Uniformly Loaded	264
10-4	A Vehicle Crossing a Flexible Span	268
10-5	Maximum Midspan Deflection of a Flexible Beam When Vehicle Is at Midspan	273
10-6	Maximum Midspan Amplitude of Vibratory Motion of Flexible Beam After Vehicle Has Crossed It	276
10-7	Maximum Midspan Deflections due to a Series of 1260kg Vehicles Crossing a 30.48m Span	288

10-8 Guideway Mass per Unit Length Required to Meet Stress and Ride Comfort Criteria 291
10-9 Span Length for Minimum Guideway Cost 308

List of Tables

4-1 Average Trip Lengths 67

4-2 Computation of the Required Number of Occupied Vehicles 71

4-3 Computation of Average Trip Length in Five-Loop Network 85

4-4 Computation of Average Trip Length in Thirteen-Loop Network 86

4-5 Classification of Transit Systems 91

5-1 Geometric, Performance, and Cost Parameters for a Typical Network System 120

6-1 Mobility in Various Cities 135

6-2 Fraction of Area Reachable without Transfer 150

7-1 Components of Minimum Headway for $t_e > t_f$ 165

7-2 Minimum Headway and Maximum Collision Velocity in Sudden-Acceleration Failures 170

Foreword

Urban transportation, as Professor Anderson cites it in his introduction, "refers to the totality of movement within an urban area by public and private means." Thus, urban transportation consists of transit, which refers to the process of transferring goods and people in urban areas by public conveyances; and private conveyances, which almost always move on publicly financed roads. This book addresses itself to transit systems theory and in so doing, fills a void which has existed for decades and provides a means for the organized presentation of principles for study and seeking of solutions.

It is safe to state that there has been more written and said about urban transportation and transit systems, particularly, since the advent of participation by the Urban Mass Transportation Administration in urban transportation matters than ever before. There is a good reason for this. Historically, urban transit systems, particularly in the United States, were a private business and a very highly successful business until the overwhelming success of the private automobile caused the changes in the style of living of urban America which in turn resulted in the collapse of urban transit as a viable and profitable business enterprise. Urban transit even under private ownership has always performed a public service. As the privately owned transit systems were increasingly taken over by public bodies, however, the fundamental nature of transit has changed. Thus, today it is looked upon as a vital public service whose level of service is determined by societal needs within limits of societal means. It is this public service nature of urban transit, which puts it in competition for local resources with health, education and welfare and other societal services, that causes much of the difficult controversy surrounding transit. It is partly because of this competition for public funds that frequently it is observed that transit is not a problem amenable to technological solutions, but rather a matter for institutional and financial solutions. Nevertheless, there are many of us who cannot accept the proposition that, in the last quarter of the twentieth century, technology is incapable of providing some solutions, if not all the answers, to frequently debated urban transportation problems. The fundamental modes of conventional transit have been around for a half to three quarters of a century and some of them even longer. It is only since the 1950's, and predominantly since the 1960's, that modern technology and those pursuing modern technology are increasingly focusing and seeking technological solutions to transit problems.

While this book addresses many forms of urban transit, it by design emphasizes the network characteristics of transit and focuses heavily on automated transit systems. Technology can undoubtedly contribute to each and every mode of transportation, including urban transit, by improving componentry, reducing cost of operations, and improving performance. Yet, automation is probably the only new technology which can potentially contribute to revolutionary improvements in transit performance.

Automation through the evolution of electronics, digital computers, systems analysis and systems technology is indeed the product of the mid-twentieth century. While it has penetrated life all around us, it has made relatively few inroads into the operations of urban transit systems. There is good reason for this. Automation achieved its most spectacular initial successes with the military and in aerospace. In all those applications, the perceived benefits over the cost of automation are usually extremely high and therefore automation was readily accepted. Because of very high perceived benefit-cost ratios, even the initial difficulties in achieving high reliability through automation were accepted, or better, circumvented by the generous application of redundancies to assure any required mission success criteria. Automation, however, also succeeded in commercial, civilian business. It surrounds us in the form of airlines reservation systems, automated communications in our telephone networks, business data processing, check clearances, bank and insurance applications as well as manufacturing process control. These are all areas in which automation has produced such a significant quantum jump in producitivity that the occasional failure of automation was readily tolerated because the time lost due to repairing the failed componentry was quickly made up by the high productivity of the automated process.

Transit may be unique in this respect. It is unlike the military or aerospace class of activities, where one can afford sufficient redundancies to achieve a high probability of success and where, in the interest of national security, or national prestige, economic considerations are not as critical as in other areas. On the other hand, transit cannot be compared with the manufacturing process control or reservations or banking systems of civilian undertakings because in transit, the temporary loss of service, if it occurs frequently, cannot be tolerate by the citizens. The most important performance characteristic of transit is probably its dependability to carry people reliably to their destination. Thus, transit may be the highest challenge to the introduction of automation because, on one hand, one cannot afford the redundancies customary in military and aerospace application while, on the other hand, one cannot afford the occasional downtimes associated with the off-line functions that automation usually performs in business. To meet automated operations dependably and at affordable costs is then the challenge to automated transit in urban applications.

Professor J. Edward Anderson understands this challenge and it is not by accident that he devotes such significant segments of this book to life cycle cost, theory of reliability allocation, redundancy, and failure mode and effect analysis. The reader might have different ideas and conclusions about the relative merits of a variety of approaches to automated transit systems, but nobody can quarrel with the need for the concept expressed in the title of Chapter 11, "Design for Maximum Cost Effectiveness."

It is because of this deep understanding of the environment in which automated transit needs to be deployed and can provide its contribution that Professor Anderson's book is highly recommended not only for students of modern technology but also for all of us who are interested in improving urban transportation.

G.J. Pastor
Associate Administrator for Technology
Development and Deployment
Urban Mass Transportation Administration

Preface

The past decade has witnessed a revival of interest in alternative means for movement of people in cities. Hundreds of schemes have been proposed, dozens of which have reached the test-track stage, and a few of which are in operation. Debate over alternative concepts has been intense and heated, indicating the strength of feeling many people have about the subject. All too often, based on inadequate analysis, a great deal of money has been spent on concepts which are later found to be of limited utility. There has been a need to develop a theoretical foundation for the analysis and synthesis of transit systems in a form that can be taught in schools and assimilated by practicing engineers, many of whom enter the transit field with no formal training in the subject. This book is offered as a step in the fulfillment of such a need. It is written as an engineering textbook with sufficient material for a one-year course, and should be understandable to persons with the background in mathematics and the physical sciences usually attained by students in the senior and first-year graduate levels of engineering. While the emphasis is on the technical aspects of transit, it is important to keep in mind that transit is an interdisciplinary subject and that a rounded transit engineer needs to understand many subjects beyond the scope of this book.

For reasons stated in the introduction to chapter 11, most but not all of this book pertains to the theory of automated guideway transit. There has been some discussion of standardization of these systems, but before systems can be standardized they must be classified and the relative merits of each system must be understood. But even then each of the classified systems can be cost effective to a greater or lesser degree depending on how its variable properties are chosen. Before standardization makes sense, the optimum parameters must be known, that is, those parameters that result in maximum cost effectiveness, usually measured by the cost per trip or per kilometer-trip. In a recent study of automated guideway transit systems by the Office of Technology Assessment, it was proposed that AGT systems be classified as either shuttle-loop, group rapid transit, or personal rapid transit. Unfortunately, the first of these refers to the geometry of the lines and the second and third to the service characteristics. Classification into just three types may have apparent advantages for policy makers, but it is much too simplistic for detailed understanding of AGT systems. As a start the classification matrix presented in the summary of chapter 4, which results in identification of twenty-five types of systems, is suggested. Because of my association with the International Conferences on Personal Rapid Transit, I am usually identified with that concept. Nonetheless, in this book the term personal rapid transit is used only in references. The reason is that the term has been used to identify too wide a range of systems, many of which are very cost ineffective. As a result, the term has in my opinion become worse than useless in that its utterance usually generates more heat than light.

My purpose is and has been to search by rational analysis supported by a factual basis for characteristics of and parameter choices within transit systems that will make it possible to build these systems in such a way that the public can be given the greatest service for the least money consistent with environmental requirements. The best way I know to measure "the greatest service for the least money" is by the total cost per trip, and I don't believe the full potential of transit can be realized until systems that minimize the cost per trip become available. It is not surprising that such a quest, joined by many people all over the world, has resulted in systems radically different from those in operation; nor, because of the fundamental importance of transit and the many specialties within it, is it surprising that new types of systems are resisted. Progress can be made only as the fundamentals of the subject become more widely understood.

Hundreds of people have contributed directly and indirectly to this book through their papers and reports, and through conversations I have been privileged to have with them. Mainly as a result of the aforementioned conferences, I have been able to see and read about the contributions of people to the subject all over the globe. The development of much of the material began at the University of Minnesota under the sponsorship of a grant from the Minnesota State Legislature, without which little could have been done. Grants from the Urban Mass Transportation Administration also contributed needed support. The bulk of the work was completed while the author was on leave first with the Colorado Regional Transportation District, and then with the Raytheon Company in Bedford, Massachusetts. Upon returning to the University of Minnesota, the author used much of the material in a two-quarter course, where it was improved by student comments. The moral support and encouragement of many laypersons, whose stake in improving transit can come only with improved transit service, has been essential, and much helpful advice and encouragement has come from many of the members of the Advanced Transit Association. Finally, without the patience and understanding of my wife, Cindy, this book could not have been written.

1

Introduction

Transit is a word of many meanings. As used in this book, it refers to the process of transporting people and goods within urban areas by public conveyances. The term ''urban transportation'' is used in contemporary literature to refer to the totality of movement within an urban area by public and private means, even though the private conveyances must almost always move on publicly financed roadways. The term ''transit system'' refers in this book to all of the hardware needed to provide the function of transit. The hardware may include vehicles, roadways or guideways, stations, and central facilities for operation and maintenance. Transit systems theory is the underlying system of general principles of design, operation, and performance that provide a reasoned basis for selection of specific characteristics and parameters of transit systems. No author can claim to set in print all of transit systems theory but one can hope to pick up where others have left off and present such a body of knowledge in a more general and consistent form. Transit systems theory cannot be developed in a vacuum, but only after the development, operation, and public evaluation of many types of transit systems over a period of many years. As a parallel, the technology of heat engines developed on an ad hoc basis for many decades before the science of thermodynamics led to a fundamental understanding of the thermal processes within the engine and from that to a marked improvement in the performance and efficiency of heat engines.

The beginnings of transit occurred in the early part of the nineteenth century with horse-drawn streetcars[1], a forerunner of which was the stagecoach, which was limited in weight and size because of the condition of the roadways. By operating on a guideway of steel rails instead of mud roads, a team of horses could pull a load many times as great at higher speeds. Because it permitted the cost of the horse and driver to be amortized over many more patrons, and it decreased the trip time, the horse-drawn streetcar became very popular. With the advent of the electric motor and the central-station dynamo later in the nineteenth century, it was natural to electrify the streetcar; however, the history of this development shows many failures before the electric streetcar of the twentieth century emerged. A problem with the streetcar, which became more and more severe in dense cities, was its interference with other traffic, which resulted in slow operating speeds and many accidents. This problem could be solved with then-existing technology by building exclusive rights-of-way for the tracks usually either overhead or in tunnels. The concept of rapid rail transit was born and in many large cities became the backbone of the transit system.

Early in the twentieth century, the technology of the heat engine had developed sufficiently to be used to propel carriages, and these were refined and manufactured in ever increasing numbers. With sufficient numbers of the evolving automobile in use, public support increased for better roads. Once the roads and vehicles had improved sufficiently, the original reason for track-bound street cars faded and the transit bus took over its function in more and more cities, until in the mid 1950s the streetcar had all but disappeared. Earlier, in the first two decades of the twentieth century, the need for a more flexible form of transit than the streetcar or rapid rail was met by advancing automotive technology with the jitney, a semi-demand-activated large automobile or bus that picked up and dropped off people along an approximate route. The jitney competed so successfully with the streetcar that the owners of the large and politically powerful streetcar companies succeeded in persuading legislators to pass laws banning it. Operating small vehicles in a demand mode was considered unfair competition for the less flexible trackbound vehicles, and they were permitted to remain only in the form of the taxi, which is too expensive for most people to use for daily travel. The free market system was not permitted to function to allow the most competitive form of transit to evolve.

In the 1930s and 1940s, many people dreamed of owning automobiles because of the complete flexibility of movement, comfort and privacy they provided but could not afford them. During the 1950s, however, increasing affluence and low cost housing loans led to the complete dominance of the automobile as the mode of urban transportation in most cities in North America and to the spread city of today. Public transit could no longer compete and one after another transit companies went bankrupt. By the early 1960s the increased numbers of automobiles and the still present need for transit regardless of cost combined to initiate the revival of transit by Congressional action, which established the Urban Mass Transportation Administration. While a paragraph was included in the law directing UMTA to investigate the promise of totally new types of transit systems, the main driving force behind its creation was evidently the view that revival of the fixed guideway systems of old with modern engineering refinements would solve the problems brought on by dominance of the automobile. In spite of UMTA funded work which showed that a gradual reintroduction of systems of the past would not prevent continued worsening of congestion, the vast bulk of federal funds were invested in conventional systems. A decade and a half later, these conclusions, summarized by Hamilton and Nance[2], seem generally correct. New ideas are still needed. Perhaps the frustration of rising deficits and disappointing performance will increase interest in innovation, notwithstanding early experiences.

In developing theory of transit systems, this book builds on a great deal of activity in development of new transit systems made possible by a wide variety of technological advances since World War II[3]. Theory is developed, not only of existing systems, but of new systems by considering the "transit system" as a field of initially undetermined characteristics and parameters subject to a field of requirements coming from analyses of the needs of the public. On examination of the results of dozens of transit system development programs, it is clear that most have failed or will fail because characteristics or parameter choices were made on the basis of unsubstantiated but plausible assumptions. People have dreamed for perhaps as long as they have populated the earth of better means of getting to where they want to go. In recent times people have dreamed of and have invested money in many transport ideas, all but a few of which have proved or will prove to be impractical because of the high cost per ride, or because of another fault, which if corrected leads to high cost per ride. Unsupported intuition has provided much misguidance in developing new transit systems which will at a sufficiently low cost meet needs and expectations of the public. Transit systems theory is needed to find optimized solutions based on serving the public as well as possible for the least money subject to environmental and performance constraints.

This book presents basic areas of transit systems theory applicable to a wide variety of types of transit systems. In the final chapter, the previously developed theory is synthesized into characteristics of transit systems optimized to the extent permitted by the knowledge obtained. In chapter 2, basic performance relationships used over and over again in transit systems analysis are developed. These refer to the longitudinal motion of vehicles, and involve limits on acceleration and rate of change of acceleration (jerk) permissible based on the criterion of human comfort in normal and emergency circumstances. The limits used are in the range generally accepted; however, insufficient testing has been done to establish these firmly for all classes of riders. Therefore, the reader should keep in mind the basic algebraic relations in making computations for specific systems. Chapter 3 deals with similar requirements for lateral motion but here these requirements lead to the specification of curvature limits for guideways in various practical situations. Chapter 4 then builds on previous work in development of geometric and performance relationships for various types of transit systems classified as indicated in its summary. In chapter 5, general cost equations are developed for all types of transit, cost effectiveness relationships are developed and discussed, and the general formulas are applied to a field of specific types of systems. In chapter 5, patronage is a parameter. It is important to use patronage this way in initial calculations to give the analyst and policy maker a good representation of the variation of the cost effectiveness parameters with patronage. Then in chapter 6 the subject of patronage analysis is introduced in enough detail to give the systems engineer a good feeling for the subject, but not in the exhaustive detail needed for specific recommendations. Patronage analysis is the heart of the whole transit problem for it deals directly with factors that measure the attractiveness of specific transit features to the potential transit-riding public. It is behavioral, however, and beyond the professional competence of most engineers. Consequent superficial treatment of the subject is the probable cause of failure of many transit concepts.

The three remaining subjects in chapters 7, 8 and 9, and 10, can be studied in any order. Chapter 7 develops the theory of safe operation and leads to specific performance limitations and recommended vehicle features. Chapters 8 and 9 develop a new theory of reliability requirements and reliability allocation based on minimization of life cycle cost subject to the constraint of a given level of service availability. It results in specific recommendations for equipment needed to insure adequate service availability in the systems discussed and quantifies the changes in system reliability associated with changing equipment and equipment parameters. Chapter 10 considers the problem of optimization of the characteristics of elevated transit guideways in such a way that cost per unit length is minimized. Finally, as mentioned above, in chapter 11 the previous theory is used to synthesize the transit systems characteristics that minimize the cost per trip.

The title of this book is *Transit Systems Theory*, not *The Theory of Transit Systems*, because it is not all inclusive. Other topics such as detailed patronage analysis techniques, the theory of control, operational analysis of station and interchange flows, and the theory of large-scale network simulations could and perhaps should be included in such a work. The author believes, however, that the topics included form a fundamental background useful to all transit systems engineers, and that, for the most part, beyond these the topics become specialized and can be pursued in the periodical literature. The book provides the basis for determining what should be controlled, but it leaves to others the detailed implementation of control.

Notes

1. A more detailed and illustrated history of transit development is given in the *Lea Transit Compendium,* Vol. 2, No. 1, 1975, published by the N.D. Lea Transportation Research Corporation, 123 Green Street, Huntsville, Alabama 35801.

2. William F. Hamilton and Dana K. Nance, "Systems Analysis of Urban Transportation," *Scientific American,* Vol. 221, No. 1, July 1969.

3. Much of this work can be found summarized in three volumes of papers: *Personal Rapid Transit, Personal Rapid Transit II,* and *Personal Rapid Transit III,* distributed by the Audio Visual Library Services, University of Minnesota, 3300 University Avenue S. E., Minneapolis, Minnesota 55414. The most comprehensive earlier work on the theory of transit systems known to the author appeared in a series of reports published between 1969 and 1972 on the Cabtrack System by the Royal Aircraft Establishment, Ministry of Defence, Farnborough, Hants, England. Unfortunately, these reports have never been released for general circulation. The first post World War II book that gives a systematic presentation of transit concepts and leads to conclusions in general agreement with those of this book is *Individualized Automatic Transit and the City* by Donn Fichter, 1430 East 60th Place, Chicago, Illinois 60637.

This page is intentionally blank.

2

Basic Performance Relationships

In transit systems analysis the need continually arises to relate kinematic characteristics such as trip time, time to stop, trip length, and stopping distance to line speed, maximum acceleration, and maximum jerk. These relationships are derived and presented in this chapter for future reference.

In deriving the kinematical relationships, it is necessary to make use of the experimental fact that, for comfort of the riders, the ratio of jerk to acceleration should not exceed unity in units of seconds. Stated in another way, the acceleration should not build up to its maximum value or decrease from its maximum value to zero in less than one second. Occasional use is also made of the generally accepted value of maximum service acceleration of about one eighth times gravity for standing-passenger vehicles, and one quarter times gravity for vehicles in which all passengers are seated.

Because they follow directly from the kinematical relationships, relationships for acceleration power and average energy per trip are derived and presented in this chapter.

2.1 The Acceleration Profile

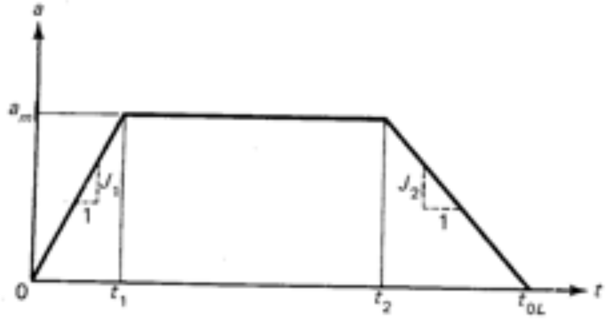

Figure 2-1. The Acceleration Profile

Consider the acceleration of a vehicle from rest to line speed V_L. The maximum acceleration during the maneuver is a_m. Consideration of human comfort requires that a_m be obtained in a finite time and at a maximum rate J_1, called the jerk. As the vehicle approaches line speed, the acceleration a_m is caused to diminish to zero at a finite rate J_2, which may, for reasons discussed in Section 2.3, not equal J_1.

For mathematical convenience, the acceleration profile just described is assumed to be composed of a series of straight lines as shown in figure 2-1. This is an idealization of an actual acceleration profile, which is continuous in its derivatives, because forces, and even rates of change of forces, cannot be applied in zero time.

The area under the acceleration-time curve from $t = 0$ to t is the velocity at time t. Thus the velocity at time t_1 is

$$V_1 = V(t_1) = \tfrac{1}{2} a_m t_1 \tag{2.1.1}$$

But

$$t_1 = \frac{a_m}{J_1} \tag{2.1.2}$$

Therefore

$$V_1 = \frac{a_m^2}{2J_1} \tag{2.1.3}$$

Similarly, the velocity at time t_2 is

$$V_2 = V_1 + a_m(t_2 - t_1) \tag{2.1.4}$$

and, by analogy with equation (2.1.3),

$$V_L - V_2 = \frac{a_m^2}{2J_2} \tag{2.1.5}$$

In analogy with equation (2.1.2)

$$t_{0L} - t_2 = \frac{a_m}{J_2} \tag{2.1.6}$$

Combining equations (2.1.2) through (2.1.6), we have

$$t_{0L} = \frac{V_L}{a_m} + \frac{a_m}{2J_1} + \frac{a_m}{2J_2} \tag{2.1.7}$$

If $J_1 = J_2 = J$, equation (2.1.7) takes the easily remembered form

$$t_{0L} = \frac{V_L}{a_m} + \frac{a_m}{J} \tag{2.1.8}$$

The time t_{0L} can be interpreted as either the time requred to reach speed V_L from rest, or by symmetry the time required to stop from speed V_L.

The jerk J should be high as possible to minimize t_{0L}, but comfort considerations dictate that J be less than or equal to a_m in seconds units. Thus, the contribution of a_m/J to t_{0L} is usually about one second, usually small compared to V_L/a_m.

2.2 The Velocity Profile and Stopping Distance

The curve of figure 2-2 is the integral of the curve of figure 2-1. The area under it is the distance travelled. In the region from $t = 0$ to $t = t_1$, the acceleration is

$$a = J_1 t$$

the velocity is

$$V = ½ J_1 t^2$$

and the distance travelled is

$$D = \frac{J_1 t^3}{6}$$

Substituting equation (2.1.2)

$$D(t_1) = D_1 = \frac{a_m^3}{6J_1^2} \qquad (2.2.1)$$

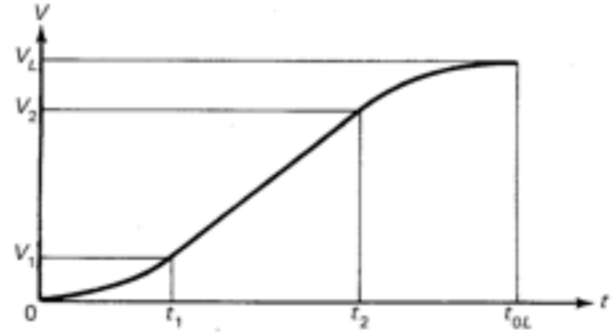

Figure 2-2. The Velocity Profile

By analogy, and using equation (2.1.6),

$$D_{0L} - D_2 = V_L(t_{0L} - t_2) - \frac{a_m^3}{6J_2^2}$$

$$= \frac{a_m}{J_2}\left(V_L - \frac{a_m^2}{6J_2}\right) \qquad (2.2.2)$$

From figure 2-2, the area of the trapezoid between t_1 and t_2 is

$$D_2 - D_1 = \frac{(V_1 + V_2)}{2}(t_2 - t_1)$$

Substitute for $t_2 - t_1$ from equation (2.1.4) and multiply the expression out. Then substitute for V_2 from equation (2.1.5) and for V_1 from equation (2.1.3). The result is

$$D_2 - D_1 = \frac{1}{2a_m}\left[\left(V_L - \frac{a_m^2}{2J_2}\right)^2 - \left(\frac{a_m^2}{2J_1}\right)^2\right] \qquad (2.2.3)$$

Adding equations (2.2.1), (2.2.2), and (2.2.3), we have

$$D_{0L} = \frac{V_L}{2}\left(\frac{V_L}{a_m} + \frac{a_m}{J_2}\right)$$

$$- \frac{a_m^3}{24}\left(\frac{1}{J_2^2} - \frac{1}{J_1^2}\right) \qquad (2.2.4)$$

in which equation (2.1.6) has been substituted.

Following equation (2.1.8) it was indicated that under usual circumstances J is approximately equal to a_m. The maximum value of a_m considered acceptable from the standpoint of comfort is about 2.5m/s^2 or 0.25 gee. Therefore a_m^3/J^2 is approximately 2.5, and the term in equation (2.2.4) proportional to a_m^3 contributes no more than 10 cm to D_{0L}. Therefore, to a good approximation,

$$D_{0L} = \frac{V_L^2}{2a_m} + \frac{V_L a_m}{2J_2} \qquad (2.2.5)$$

The value D_{0L} given by equation (2.2.5) is the distance the vehicle travels while its velocity changes by V_L with the indicated values of acceleration and jerk. The word "changes" is used to emphasize that the result is the same if the transition is from rest to line speed or from line speed to rest, if the change in velocity is V_L. Thus equation (2.2.5) can be referred to as the stopping distance. In the case of deceleration, however, the problem of power limitation, discussed in section 2.3, does not exist and we can set $J_1 = J_2 = J$. Then

$$\text{Stopping Distance} = \frac{V_L^2}{2a_m} + \frac{V_L a_m}{2J} \qquad (2.2.6)$$

$$= \frac{V_L}{2}\left(\frac{V_L}{a_m} + \frac{a_m}{J}\right)$$

2.3 Acceleration Power

Power is force times velocity. The acceleration force $F = ma$, where m is the mass of the vehicle and a is the acceleration. Thus

$$\text{Acceleration Power} = P_a = maV \qquad (2.3.1)$$

The energy required to accelerate an object from rest to velocity V is

$$\text{Energy} = \int_0^t P_a \, dt = m \int_0^t aV \, dt$$

But $a = dV/dt$. Therefore

$$\text{Energy} = m \int_0^V V \, dV = \frac{mV^2}{2} \qquad (2.3.2.)$$

the well-known formula for kinetic energy.

In accelerating a transit vehicle, we are interested in the maximum power required to overcome inertia, air drag, and road resistance. This will be dealt with in more detail in section 2.6, but here we concentrate on acceleration power given by equation (2.3.1) to determine how power limitations effect D_{0L} and t_{0L}. The product aV increases linearly from t_1 to t_2 in figure 2-1 and then must fall off to zero at t_{0L} where $a = 0$ after possibly exceeding the value at t_2. It can be shown, however, that unless V_L is less than 1.5 a_m^2/J_2, aV reaches its maximum value at t_2. This is assumed in the following paragraph.

The reason for possibly making J_2 less than J_1 is to limit the power required and hence the size of the motors. In this circumstance, we can assume that a_m remains constant until the power reaches $P_{\max}$, following which the acceleration is reduced to zero at rate J_2. In this case, using equation (2.2.5),

$$P_{\max} = ma_m V(t_2) = m\left(a_m V_L - \frac{a_m^3}{2J_2}\right) \tag{2.3.3}$$

in which equation (2.1.5) has been used. If the maximum available power for acceleration is known, equation (2.3.3) can be used directly to compute the maximum permissible value of J_2.

To determine the effect of power limitations on D_{0L} and t_{0L}, consider the following changes due to reduction in J_2:
From equation (2.3.3)

$$\Delta P_{\max} = -\frac{ma_m^3}{2}\Delta\left(\frac{1}{J_2}\right) \tag{2.3.4}$$

From equation (2.2.4)

$$\begin{aligned}\Delta D_{0L} &= \frac{V_L a_m}{2}\Delta\left(\frac{1}{J_2}\right) - \frac{a_m^3}{24}\Delta\left(\frac{1}{J_2^2}\right)\\ &= \frac{a_m}{2}\left[V_L - \frac{a_m^2}{12}\left(\frac{1}{J_1} + \frac{1}{J_2}\right)\right]\Delta\left(\frac{1}{J_2}\right)\end{aligned} \tag{2.3.5}$$

From equation (2.1.7)

$$\Delta t_{0L} = \frac{a_m}{2}\Delta\left(\frac{1}{J_2}\right) \tag{2.3.6}$$

For purposes of rough estimates, let

$$P_{\max} \approx m a_m V_L \qquad D_{0L} \approx \frac{V_L^2}{2a_m} \qquad t_{0L} \approx \frac{V_L}{a_m}$$

Then, if we divide equation (2.3.5) by equation (2.3.4), the result can be written in the form

$$\frac{\Delta D_{0L}}{D_{0L}} = -2\,\frac{\Delta P_{\max}}{P_{\max}}\left[1 - \frac{a_m^2}{12V_L}\left(\frac{1}{J_1} + \frac{1}{J_2}\right)\right] \tag{2.3.7}$$

and, if we divide equation (2.3.6) by equation (2.3.4), the result can be written in the form

$$\frac{\Delta t_{0L}}{t_{0L}} = -\frac{\Delta P_{\max}}{P_{\max}} \tag{2.3.8}$$

We see that a given percentage reduction in the maximum power increases D_{0L} by somewhat less than twice that percentage, and t_{0L} by the same percentage. This magnitude of change in t_{0L} is usually insignificant because t_{0L} is a small fraction of the total trip time. If the stations are on the main line, the indicated change in D_{0L} is not significant unless the stations are so close together that V_L can no longer be reached. Thus, with on-line stations, reduction in J_2 below J_1 is usually advantangeous. If the stations are off line, however, increasing D_{0L} directly increases the length of the acceleration ramps, thus adding directly to the cost of the system. Since reserve power is needed to operate on grades and in high winds, it is doubtful that a given percentage reduction in $P_{\max}$ will reduce overall cost enough to offset twice that percentage increase in off-line ramp cost and visual impact. Thus, in dealing with off-line station systems we will always assume $J_2 = J_1$.

2.4 Trip Time and Average Velocity

Each trip is composed of one or more maneuvers of the type depicted in figure 2-3. The vehicle begins to move at $t = 0$, reaches maximum velocity at $t = t_a$, cruises to $t = t_b$, decelerates and reaches zero velocity at t_c, waits at a station for a time t_D (called the station dwell time), and repeats its cycle. Let the station-to-station time be denoted by t_s. Then, from Figure 2-3,

$$t_s = t_D + t_a + (t_c - t_b) + (t_b - t_a)$$

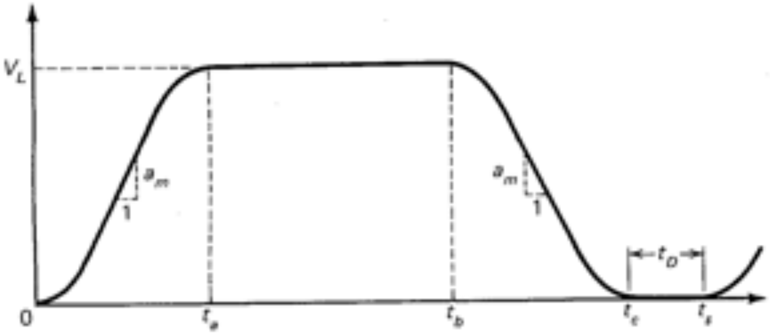

Figure 2-3. The Station-to-Station Velocity Profile

We assume that t_a is given by equation (2.1.7) with $J_1 \neq J_2$, and $t_c - t_b$ with $J_1 = J_2$. Then

$$t_s = t_D + \frac{2V_L}{a_m} + \frac{3a_m}{2J_1} + \frac{a_m}{2J_2} + (t_b - t_a) \tag{2.4.1}$$

Let D_s be the distance between stations, that is, the area under the velocity profile of figure 2-3. The distance travelled from $t = 0$ to $t = t_a$ is given by equation (2.2.4) with $J_1 \neq J_2$, and from $t = t_b$ to $t = t_c$ by the same equation with $J_1 = J_2$. The distance from $t = t_a$ to $t = t_b$ is $V_L(t_b - t_a)$ in which $(t_b - t_a)$ is given by equation (2.4.1). Thus, D_s can be written in the form

$$\frac{D_s}{V_L} = \frac{V_L}{a_m} + \frac{a_m}{2}\left(\frac{1}{J_2} + \frac{1}{J_1}\right)$$

$$- \frac{a_m^3}{24V_L}\left(\frac{1}{J_2^2} - \frac{1}{J_1^2}\right) + (t_b - t_a) \tag{2.4.2}$$

Subtracting equation (2.4.2) from equation (2.4.1) eliminates $(t_b - t_a)$ and we have

$$t_s = t_D + \frac{D_s}{V_L} + \frac{V_L}{a_m}$$

$$+ \frac{a_m}{J_1} + \frac{a_m^3}{24V_L}\left(\frac{1}{J_2^2} - \frac{1}{J_1^2}\right)$$

Suppose $J_2 = 0.5J_1$, $J_1 = a_m$, $a_m = 2.5$ m/s², and $V_L = 10$ m/s. In this case the rightmost term is only 0.03 s. Thus in practical cases we can neglect the rightmost term and obtain

$$t_s = t_D + \frac{D_s}{V_L} + \frac{V_L}{a_m} + \frac{a_m}{J_1} \tag{2.4.3}$$

It will be noticed that the form of this equation makes it very easy to remember. The trip time is simply the sum of terms like equation (2.4.3), one corresponding to each stop. If the vehicle must slow down somewhere enroute, a formula for the additional delay is given by equation (2.5.3).

The average velocity V_{av} is simply D_s/t_s. Using equation (2.4.3),

$$V_{av} = \frac{D_s}{t_D + \frac{D_s}{V_L} + \frac{V_L}{a_m} + \frac{a_m}{J_1}}$$

or

$$\frac{V_{av}}{V_L} = \frac{D_s}{D_s + \bar{D}} \tag{2.4.4}$$

in which

$$\bar{D} = V_L\bar{t}_D + \frac{V_L^2}{a_m} \tag{2.4.5}$$

and

$$\bar{t}_D = t_D + \frac{a_m}{J_1} \tag{2.4.6}$$

By comparing equation (2.4.5) with equation (2.2.6), we note that when $t_D = 0$, $\bar{D}$ is the minimum D_s which will permit speed V_L to be reached. Equations (2.4.4) and (2.4.5) are plotted in figure 2-4 in such a way that V_{av}/V_L can be read directly as a function of four variables: D_s, V_L, a_m, and t_D. If the figure is rotated 90 degrees counterclockwise, the right-hand graph is a plot of equation (2.4.5) for five values of t_D, two values of a_m, and for $a_m = J_1$. The lower value of a_m is generally accepted as the appropriate normal acceleration and deceleration for vehicles in which standees are permitted, and the higher value of a_m is the corresponding value if all passengers are seated. The five values of t_D to cover the range used in practice. Note that the five dashed curves are shifted upward from the corresponding set of solid curves.

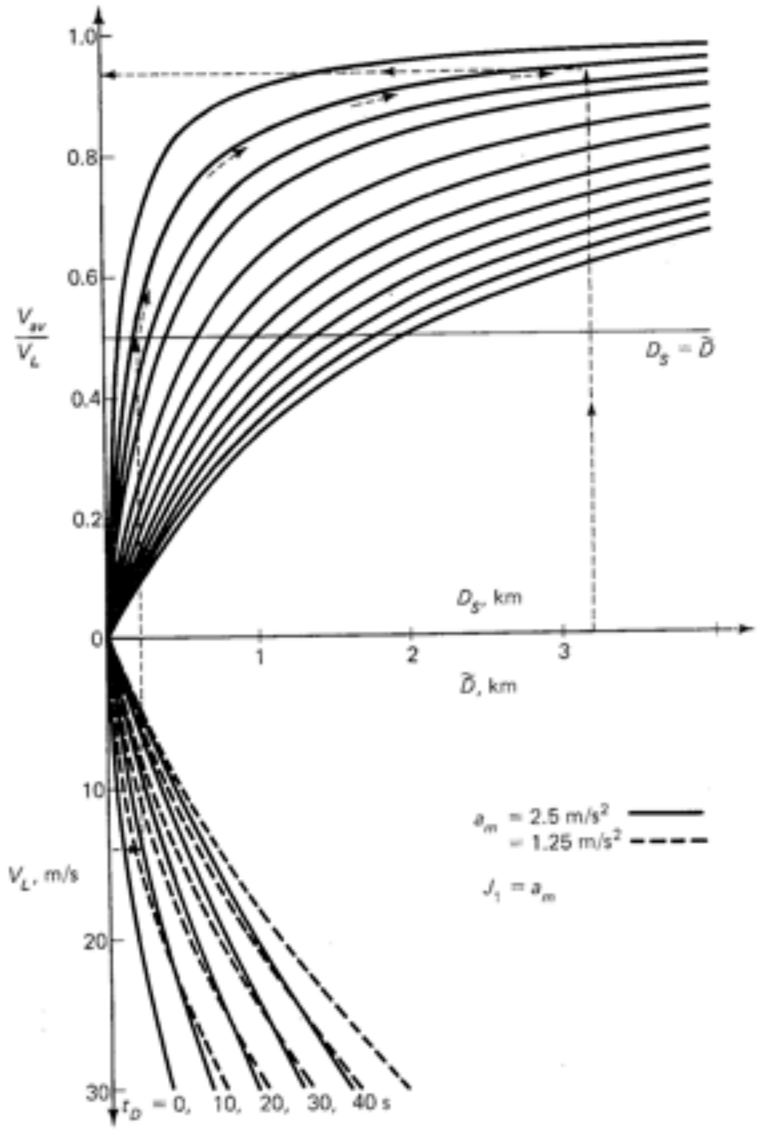

Figure 2-4. The Average Velocity

If we rotate the figure back to its original position, the upper set of curves is a plot of V_{av}/V_L on the ordinate and D_s on the abscissa, with the abscissa taken common to the ordinate of the lower curve. From equation (2.4.4) we see that $D_s = \tilde{D}$ when $V_{av}/V_L = 0.5$. Therefore connection between the lower and upper sets of curves can be made along the line $V_{av}/V_L = 0.5$. Arrows on the curves illustrate their use in an example in which the input variables in the lower set of curves are $t_D = 10$ s, $a_m = 2.5$ m/s^2, and $V_L = 14$ m/s. These values give $\bar{D} = 0.22$ km. Follow the dotted line through $\bar{D} = 0.22$ km up to $V_{av}/V_L = 0.5$. Here $\bar{D} = D_s$, therefore the solution for V_{av}/V_L lies on the curve which passes through the point $V_{av}/V_L = 0.5$, $D_s = 0.22$ km. The family of values of V_{av}/V_L for various D_s fall along this curve, outlined by arrows. For the specific value $D_s = 3.2$ km, we find that $V_{av}/V_L = 0.936$. Often V_{av} will be specified from patronage considerations. Then a family of solutions can be found by picking values of V_L and finding D_s from the graph in a similar fashion.

2.5 Time and Distance Loss due to Speed Reduction

Often it is necessary to compute the time lost in slowing from line speed V_L to a reduced speed V^*, in which the reduced speed is maintained for a distance D^* and a time interval D^*/V^*. An example is going around a curve. The situation is shown in figure 2-5, in which we assume the transition occurs with the acceleration profile shown in figure 2-1, with $J_2 = J_1$ on deceleration and J_1 greater than or equal to J_2 on acceleration. The values D_{a_1} and D_{a_2} in figure 2-5 are taken from equation (2.2.4), with the term proportional to a_m^3 neglected, by substituting $V_L - V^*$ for V_L. Thus

$$D_{a_{1,2}} = \frac{(V_L - V^*)}{2}\left(\frac{V_L - V^*}{a_m} + \frac{a_m}{J_{1,2}}\right) \tag{2.5.1}$$

[illegible]

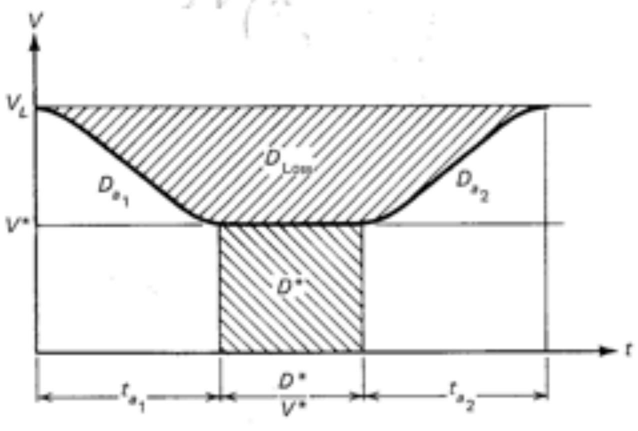

Figure 2-5. Velocity Profile in Speed Reduction

Dist to slow down from V_L to V^*

[illegible] $= \left(\frac{V_L + V^*}{2}\right)\left(\frac{V_L - V^*}{a_m} + \frac{a_m}{J_m}\right)$

The time intervals t_{a_1} and t_{a_2} are given by equation (2.1.7) with V_L replaced by $V_L - V^*$, and with $J_2 = J_1$ and $J_2 \leq J_1$, respectively. Then from figure 2-5,

$$D_{\mathrm{LOSS}} = (V_L - V^*)\left(t_{a_1} + \frac{D^*}{V^*} + t_{a_2}\right) - D_{a_1} - D_{a_2}$$

$$= (V_L - V^*)\left(\frac{V_L - V^*}{a_m} + \frac{a_m}{J_1} + \frac{D^*}{V^*}\right) \qquad (2.5.2)$$

Finally, the time loss is simply the time required to make up the distance D_{LOSS} at speed V_L. Thus

$$t_{\mathrm{LOSS}} = \frac{D_{\mathrm{LOSS}}}{V_L} \qquad (2.5.3)$$

2.6 Average Power Consumption

Consider a train of n_T vehicles, each of mass M_v and frontal area A_v, following the velocity profile of figure 2-3. The total energy input to the vehicle from $t = 0$ to t_s divided by t_s is the average power consumption. The energy input is given by

$$E(t_s) = \int_0^{t_s} \frac{FV}{\eta(V)}\, dt + n_T P_{\mathrm{aux}} t_s \qquad (2.6.1)$$

in which P_{aux} is the auxiliary power consumed per vehicle, $\eta(V)$ is the efficiency of the motors, and F is the retarding force. The force is given by

$$F = (1 - \mathscr{R})\, n_T M_v \frac{dV}{dt} + \tfrac{1}{2}\rho\, [V^2 + \langle V_w^2 \rangle]\, C_D A_v$$

$$+ n_T M_v \left[F_r(V) + g\frac{dz}{dx}\right] \qquad (2.6.2)$$

in which $\mathscr{R}$ is the energy recovery factor as a result of regenerative braking,

ρ is the air density, C_D is the coefficient of air drag, $<V_w^2>$ is the mean square wind velocity, $F_r(V)$ is the road resistance per unit mass, and dz/dx is the slope of the path. The wind velocity appears as indicated because the mean of the local wind velocity squared is

$$\begin{aligned} <(V + V_w)^2> &= <V^2 + 2VV_w + V_w^2> \\ &= V^2 + 2V<V_w> + <V_w^2> \end{aligned}$$

in which the mean wind speed relative to vehicles travelling in all directions, $<V_w>$, is zero. The road resistance term can usually be expressed adequately[1] in the form

$$F_r(V) = C_1 + C_2V \tag{2.6.3}$$

[illegible]

$$= g(a + bV)$$

If the motor efficiency is a strong function of velocity, the integral in equation (2.6.1) cannot be performed in general. However, we can always define an average efficiency $\bar{\eta}$ by the equation

$$\frac{1}{\bar{\eta}}\int_0^{t_s} FV\,dt = \int_0^{t_s} \frac{FV}{\eta(V)}\,dt \tag{2.6.4}$$

Substitute equation (2.6.3) into equation (2.6.2). Then

$$\begin{aligned} \int_0^{t_s} FV\,dt = (1-\mathscr{R})n_T\ \frac{M_V V_f^2}{2}\ + \tfrac{1}{2}\rho C_D A_V\left[\int_0^{t_s} V^3 dt + <V_w^2>\int_0^{t_s} V\,dt\right] \\ + n_T M_V\left[C_1\int_0^{t_s} V\,dt + C_2\int_0^{t_s} V^2\,dt + g\int_0^{t_s}\ \frac{dz}{dx}\,\frac{dx}{dt}\,dt\right] \end{aligned} \tag{2.6.5}$$

in which

$$\int_0^{t_s} V\,dt = D_s$$

and $dx/dt = V$.

Sufficient accuracy in the remaining two integrals can be obtained by assuming in the velocity profile of figure 2-3 that $J = \infty$. Then

$$\int_0^{t_s} V^2 dt = V_L D_s - \frac{V_L^3}{3a_m}$$

and

$$\int_0^{t_s} V^3 dt = V_L^2 D_s - \frac{V_L^4}{2a_m}$$

Substituting the three integrals into equation (2.6.5) and using equation (2.6.4), equation (2.6.1) becomes

$$E(t_s) = \frac{1}{\bar{\eta}} \left\{ (1 - \Re) n_T \frac{M_V V_L^2}{2} + ½ \rho C_D A_V \left[(V_L^2 + <V_w^2>) D_s - \frac{V_L^4}{2a_m} \right] \right.$$

$$\left. + n_T M_V \left[C_1 D_s + C_2 V_L \left(D_s - \frac{V_L^2}{3a_m} \right) + g\, z \right] \right\} + n_T P_{aux} t_s \qquad (2.6.6)$$

in which z is the change in elevation from the beginning to the end of the trip. From equation (2.4.3), t_s can be approximated by the equation

$$t_s = t_D + \frac{D_s}{V_L} + \frac{V_L}{a_m} \qquad (2.6.7)$$

Then the average power consumption is

$$P_{av} = \frac{E(t_s)}{t_s} \qquad (2.6.8)$$

in which, in the term $E(t_s)$, D_s is the average distance between stops.

2.7 Summary

Chapter 2 derives and collects basic performance equations which are used over and over again in the analysis of transit systems. These formulas are not exact because the time-position curves of vehicle motion cannot be defined precisely; however, they are developed based on idealized velocity-time curves sufficiently accurate for the purposes for which they are used. Approximations are based on generally accepted values of maximum service acceleration and jerk, and, by inference, higher order derivatives of acceleration need not be considered. The formulas derived include the time required to travel from rest to a given line speed, the stopping distance from a given line speed, the maximum power output required during acceleration, the time of a nonstop trip at a given line (or cruise) speed, the average velocity counting stops and dwells, and the time and distance lost due to a speed reduction. Since the relationship between line speed and average speed as it depends on station spacing is particularly important, it is plotted in figure 2-4. Finally, a general formula (2.6.6) for the energy per trip is developed.

Problems

1. Show that the acceleration power of an accelerating vehicle reaches its maximum at the point of transition from constant acceleration to constant jerk, if $V_L > 3a_m^2/2J_2$.
2. Consider a 10,000-kg standing-passenger vehicle moving between stops at a line speed of 30 m/s and conforming to standard comfort criteria. Compute the maximum acceleration power in kilowatts if the maximum comfort value of jerk is applied in all cases. If the power available for acceleration is reduced by 30 percent from the computed value,
   (1) by what factor must jerk be reduced as line speed is approached?
   (2) what is the penalty in increasing distance to reach maximum speed?
   (3) what is the penalty in increased time between stops?
3. It is desired to achieve an average speed of 50 km/hr in a transit system with on-line stations and standing-passenger vehicles. If the average station delay is 20 seconds, plot a curve of station spacing versus line speed. What is the minimum station spacing and at what line speed does it occur? What is the physical significance of the minimum point? If the maximum obtainable line speed is 75 km/hr, what is the minimum permissible station spacing.
4. It is desired to achieve an average speed of 50 km/hr in a transit system with off-line stations and seated-passenger vehicles. If the average trip length is 6 km and the trips are nonstop, what is the required line speed if the station delay is 20 seconds, 10 seconds?

5. In going around a right-angle curve, a seated-passenger transit vehicle is restricted to a lateral acceleration of 0.25 g. If the normal line speed is 15 m/s and the curve radius is 35 m, the vehicle must slow down in going around the curve. What is the time loss in negotiating the curve?
6. The ACME Transit Company's standing-passenger transit vehicle is to be considered in an application in which the station spacing is two km and the line speed is 80 km/hr. If the rms wind speed is 16 km/hr, the auxiliary power is 2 kw per vehicle, the propulsion efficiency is 35 percent, and the station delay is 15 seconds, what percent of the energy is saved if the cars operate in two-car trains rather than as single vehicles if regenerative braking is 50 percent effective? By what factor does energy use increase if there is no regenerative braking?

[illegible]

$M_V = 10{,}000\ kg$

C_D 1/2

$\rho = 1.2\ kg/m^3$

C_1, C_2

$A_V = 5\ m^2$

Reference

1. Thomas McGean, *Urban Transportation Technology,* Lexington Books, D.C. Heath and Company, Lexington, Mass., 1976.

$f_r = C_1 + C_2 V = g(a + bV)$

$a \approx 0.01$ (dimensionless)

$b = 6.8 \times 10^{-5}\ \frac{sec}{ft}$