# Towards a PRT capacity manual

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**Abstract**

There is an increasing awareness of Personal Rapid Transit (PRT) and its high service-quality, but one of the first issues being raised concerns PRT's capacity. Furthermore, during the PRT network design phase it is of paramount importance to know the capacity limits even before before running extensive microsimulations. However, the PRT capacity analyses is not as simple as the one of conventional public transport as it depends on many parameters, such as line-speed, vehicle protection systems, station-layouts, station operation, boarding behaviour and empty vehicle share. Instead of covering the entire problem field, this paper focuses on two important topics: (i) station-capacity comparisons of off-line, serial stations and so called "back-out stations" and (ii) the issue of line capacity bottlenecks taking into account empty vehicle flows.

The present results are thought as useful tools in the design and planning phase of a PRT system. The work may be seen as a part of a more comprehensive manual that describes PRT's capacity limits for a wider range of PRT network elements.

## 1 Introduction

Personal Rapid Transit (PRT) is an innovative public transport mode where passengers travel in small, fully automated and individually controlled vehicles. Consequently, the transport service could be direct origin-to-destination if all occupants of a vehicle are heading for the same destination.

PRT is a complex system with local and global interactions within the infrastructure and between the vehicles and the infrastructure. The infrastructure itself consists of stations and an interconnected network of guideways. Typical local interactions are: distance adjustments between two or more successive vehicles; the interaction between station and vehicle during manoeuvres and boarding processes within the station. Examples for global interactions is the vehicle routing and empty vehicle management. Global functions are usually performed by a centralized instance.

The performance of larger PRT networks can be analysed by micro-simulators which reproduce the movements of each single vehicle and the trip of all individual passengers. Micro-simulators can identify capacity bottlenecks for specific networks and for a given travel demand. However, in order to design a network, it is useful to estimate capacity limits of single network elements before passing to the micro-simulation of the entire system. In the following sections we summarize recent findings on the determination of stations-capacities, considering different station layouts and operation modes and a method to calculate the empty and full vehicle flows on a PRT network, in order to verify capacity bottlenecks.

# 2 Station capacities

In this section we will address the relation between station layouts, vehicle dynamics and the respective station capacity. It can be seen as a further development of some aspects of earlier works, see Irving, J.H (1977) and Schweizer, J. (2007).

Station capacity is defined as the maximum number of vehicles that can be loaded and unloaded per hour. Note that we do not address the problem how many passengers enter each vehicle during the boarding process. What we do consider for some station layouts is the probability distribution of the boarding time of an entire group (which may be composed of on or more passengers).

In general any PRT station consists of a deceleration line, an input-queue, an exit zone, a boarding zone, an output-queue and an acceleration line (for examples see Figs. 1 and 3) . The vehicle-loading and unloading can take place either at the same berth or at a different berth. If the station is off-line, a merge and a diverge point from the main-line may also be considered part of the station. Regarding the station-capacity, the single most significant characteristic is the number of berth where the vehicles load and unload passengers. The capacity depends also on the places where loading and unloading take place. The dimensions of all other station elements, such as acceleration lines and queuing space, can be determined in a straight forward manner as soon as the number of required berth have been determined. There are two fundamentally different PRT station layouts:

- the "classical" *serial off-line station* where vehicles line up at a platform to get loaded (see Fig. 1). A major issue with serial stations is that a loaded vehicle, which is waiting at the platform, can only depart if all vehicles in front are also loaded and ready to depart. This means the capacity does depend on the passenger's boarding times.
- the so called *back-out station* where each loading berth can be accessed individually by the vehicles (see Fig. 3). The advantage of this station configuration is that it depends less on the boarding behaviour of passengers.

As these two station types have different loading and vehicle forwarding processes, their capacity-analyses will be treated in separate sub-sections.

## 2.1 Serial stations

The boarding process of serial stations is that vehicles move into the station and line up along the platform, passengers exit the vehicle, new passengers board and each vehicle will move out of the station as soon as all vehicle in front have terminated boarding. There are basically two different strategies on how the boarding can be handled, as shown in Fig. 1.

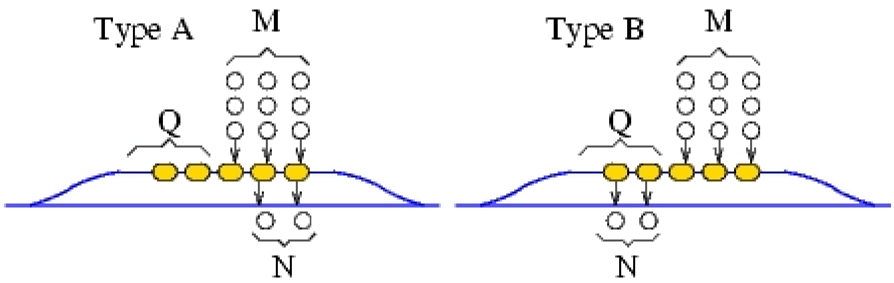


Fig. 1: Serial off-line stations with M=3 parallel load-berths, an input queue Q but two different boarding strategies. Type A: M loads and N unloads happen at the same vehicle position. Type B: unloading in unload zone before advancing to the load-berths in load zone. In general the input buffer size Q should be greater than M.

We make the distinction between Type A where passengers do exit and enter while the vehicle remains in the same berth and Type B, where unloading happens only in unload-berth and the loading happens at a load-berth in the loading area. In general we consider M load berths and N unload berth. Q is the number of vehicle slots in the vehicle input-buffer. For the capacity analyses we make the following assumptions:

- There are at all times sufficient vehicles in the stations vehicle buffer Q.
- There are always passengers at the platform waiting for a vehicle, this means no vehicle will leave the station empty.
- The boarding time of a group of passenger is in average greater than the exit time.
- The number of vehicles loading passengers is equal or greater than the number of vehicles being unloaded.
- In case of station type B, the number of places in the vehicle buffer Q is greater than the number of unload-berth N. This means the vehicle buffer can be used as unload zone as the passengers can exit while vehicles are queuing. This assumption is consistent with the above assumptions.
- The number of places in the vehicle buffer is greater than the number of load berth (Q>M). This is in any case a requirement in order to reduce the probability that the vehicle buffer runs empty, see Schweizer J. (2007).
- Boarding and exit times are Weibull-Gumbel distributed. This is a particular probability distribution which is well suited to represent boarding time characteristics, see Fig. 2.

When assessing the capacity of a serial station, the problem is that the number of loaded vehicles per hour depends on the number of unloading vehicles per hour. If, for instance, the network has a station where passengers mainly board, then the designer is interested in the maximum number of loaded vehicles per hour only, without considering unloads. This is why we focused on determining the number of loaded vehicles per hour as a function of the number of unloaded vehicles per hour, where the number of unloads is always assumed inferior or equal to the number of loads. In this way the capacity can be verified for any loads/unloads ratio greater than one.

A direct consequence of the above assumptions, is that the number of load berth M is equal or greater then the number of needed unload berth N. At station type A, where M berth are available for both, boarding and exiting, the N unload-berths is simply the average number of berth used for unload, but has no practical meaning for this station type (but N needs to be determined for analytical purposes, as we shall see below).

In order to derive an analytical expression for the station capacity and considering the above assumptions we find the minimum service time $T_S$ for one load process: The service time $T_S$ is composed by the total load/unload time $T_L$ which is required to unload $N$ and load $M$ passengers, and the forward time $T_F$ to forward a complete platoon of $M$ vehicles. As we assumed a saturated queue $Q$, we make sure that during the service time $T_S$ , a platoon of $M$ vehicles have been accumulated. In this case, the station capacity is given by

$$C_S = \frac{3600M}{T_S} = \frac{3600M}{T_L + T_F}. \tag{1}$$

Forwarding time $T_F$, and load time $T_L$, depend on passengers as well as system characteristics and will be developed in the following paragraphs.

Given a the PRT system with a maximum comfort acceleration $a_C$, a maximum velocity within the station platform $v_S$ and the berth length L which corresponds to the vehicle length $L$, we obtain for the platoon forwarding time from one of the following expressions

$$T_F = \frac{v_S}{a_C} + \frac{ML}{v_S} + M\mathrm{T}_\mathrm{D} \quad \text{for} \quad \frac{v_S^2}{a_C} < ML \tag{2a}$$

$$T_F = 2 \cdot \sqrt{\frac{ML}{a_C}} + MT_\mathrm{D} \quad \text{for} \quad \frac{v_S^2}{a_C} \geq ML \tag{2b}$$

where $T_D$ is an additional technical delay time that occurs between the start of one vehicle and the start of the successive vehicle. If all vehicles could be moved simultaneously $T_D$ equaled zero seconds. But in real systems, it can expected to be in the range of a second. The delay time $T_D$ can also be used to model jerk adaptation as jerk-rates are not considered in Eqs. (2).

The average load time $T_L$ is more difficult to estimate as it is a stochastic quantity that depends on the probability distribution of boarding- and exiting-times of the passengers. However, an analytical expression for $T_L$ can be obtained as an average of the maximum loading- and unloading times, assuming a mean boarding time $T_B$, an average unloading time $T_U$ and Weibull-Gumbel distribution with variance $V_B$ for both, boarding and exiting-times. The probability density function pdf($t$) of the Weibull-Gumbel random boarding time $t$ with average $T_B$ and variance $V_B$ is given by

$$\mathrm{pdf}(t) = \alpha \ \exp[-\alpha\,(t-\beta)] \cdot \exp[-\exp(-\alpha\,(t-\beta))], \alpha = \pi \big/ \sqrt{6V_B}, \beta = T_B - \frac{0.577}{\alpha}$$

The average boarding time $T_B$ and variance $V_B$ depend on the passenger characteristics and need to be calibrated. Examples of bourding times for different fast and slow boarding passengers are shown in Fig. 2.

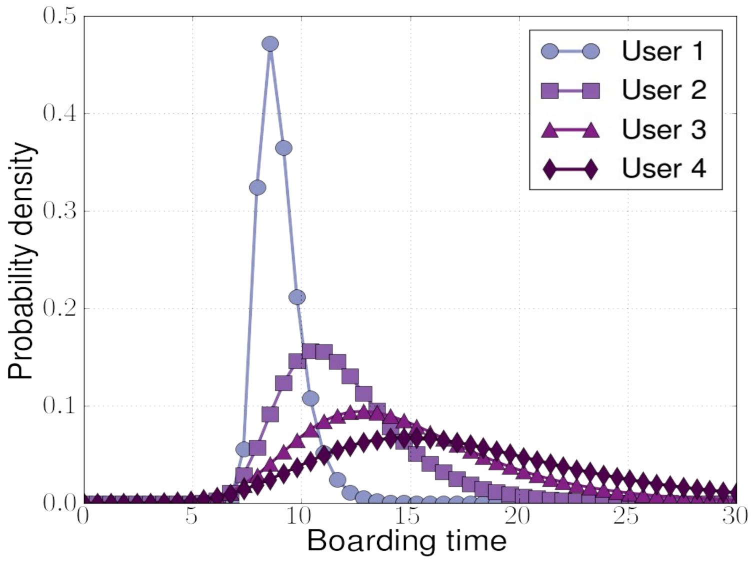


Fig. 2: Examples of boarding times, modelled by a Weibull-Gumbel probability density function pdf($t$) The boarding characteristics of the 4 passenger types differ in mean values and variance of the Weibull Gumble distribution. User type 1 is the fastest boarder with $T_B$=9s and variance $V_B$= 1$s^2$ , while user type 4 is the slowest boarder with $T_B$=18s and variance $V_B$= 49$s^2$ .

In absence of unloads, the minimum average load time $T_L$ of a station with $M$ load berth is determined by the expression:

$$T_L = \frac{\sqrt{6V_B}}{\pi} \cdot \ln(M) + T_B \quad . \tag{3}$$

This means in the absence of unloads the station capacity for a serial station with M load berth is given by Eq.(1) with $T_F$ from Eq. (2) and $T_L$ from Eq. (3), independent of Type A or B. We will now consider a given unload rate $U < C_S$ in number of unloaded vehicles per hour. The average number $N$ of berths used for unloading during one service cycle $T_S$ is determined implicitly by:

$$3600 \cdot N - U \cdot \left(T_F + T_L(N)\right) = 0 \tag{4}$$

The nonlinear, static function that determines the load time $T_L(N)$ can be derived for the two station types in Fig.2. For Type A the average load time becomes:

$$T_L(N) = \frac{\sqrt{6V_B}}{\pi} \ln\left[ N \exp\left( \frac{\pi}{\sqrt{6V_B}} (T_B + T_U) \right) + (M - N) \exp\left( \frac{\pi}{\sqrt{6V_B}} T_B \right) \right] \tag{5a}$$

while for Type B we obtain:

$$T_L(N) = \frac{\sqrt{6V_B}}{\pi} \ln\left[ N \exp\left( \frac{\pi}{\sqrt{6V_B}} T_U \right) + M \exp\left( \frac{\pi}{\sqrt{6V_B}} T_B \right) \right] \tag{5b}$$

In the presents of the given unload rate $U$, one would first compute $T_F$ from (2), then resolve the implicit equality (4) for $N$, with $T_L(N)$ from (5a or b). This may be performed by numeric root locus algorithms. Once N is know, the station capacity $C_S$ can be determined with Eq.(1) with $T_S = T_F + T_L(N)$. Numerical examples are shown in Sec. 2.3.

## 2.2 Back-out stations

With back-out stations, vehicles can be boarded more independently at the berths compared with the serial configuration. The main reason is that a loaded vehicle can back-out and depart without having to wait for the other vehicles to complete boarding (see Fig. 2).

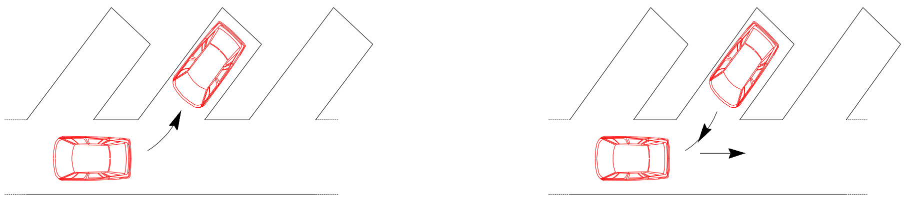

Fig. 3: An $M=3$ berth back-out station. The two figures illustrate how the vehicle moves in and out of the berth as well as from and back into the off-line track.

When analysing the capacity, it is necessary to model the station's operation procedures. Currently back-out *stations are operated asynchronously* where vehicles would move in, and out of berths, as soon as space becomes available. With the asynchronous case, arriving and departing vehicles do mix randomly along the collecting off-line track. The waiting time for a vehicle to back out, depends on the time until a free space behind the vehicle becomes available. The same applies to the time-to-enter a berth. The exact assessment of such a process is quite complex and no analytical expressions for the station capacity has been reported in literature.
However, instead of thriving for an exact solution of the asynchronous operation we can readily determine the capacity for a deterministic, *synchronous operation* which may follow the this procedure: Only half of the berth are unloading+loading, while the already loaded vehicles of the remaining berth will be moved out and filled up with new incoming vehicles, as shown in Fig. 4.

(a) (b)

Fig. 4: Illustration of synchronous load process with back-out berth. (a): impair berths get filled with incoming vehicles while berths with pair numbers unload/load. (b): now the vehicles in pair berths are ready to leave as they back out, and clear the station.

If the time intervals are known for each operational step in Fig. 4 then it is straight forward to determine the total service time $T_S$ which is required for the loading and expulsion of $M$ vehicles, just as it has been the case for the serial station. Let $T_{EN}$ be the time it takes for the vehicle to enter the berth and to come to a complete halt (see Fig. 4a). Let further be $T_{BO}$ the time that passes from starting to back out until the vehicle moves forward and cleared the berth. If $L$ is now the distance between two successive berth, then the time $T_F$ to forward a vehicle by $M$ berth can again be determined by Eqs. (2a) or (2b). This means the process shown in Fig. 4(b) lasts $T_{BO}+T_F$ and the process time to serve half of the berth is $T_{EN}+T_{BO}+T_F$ .

As this process needs to be performed for pair and impair berth numbers, the total service time $T_S = 2(T_{EN} + T_{BO} + T_F)$ and the capacity of a back-out station with $M$ berth becomes

$$C_S = \frac{3600M}{T_S} = \frac{1}{2} \cdot \frac{3600M}{T_{EN} + T_{BO} + T_F} . \tag{6}$$

However, this result, even though simple, buries two major uncertainties:

- It has not been shown that the proposed synchronous operation is the fastest way to operate a back-out type station, even though asynchronous microsimulations showed consistently inferior capacities with respect to synchronous station operation.
- The derived station capacity assumes that the slowest boarder has terminated boarding in less than half of the service time $T_S$ (see also Fig. 5). If this is not the case then the capacity will drop.

### 2.3 Numerical comparison of station capacities

In this sub-section, the previously found station capacities are compared while using identical PRT system parameters: the station speed has been limited to $v=2.77m/s$, the comfort acceleration is $a_C=1.5m/s^2$, the additional delay time $T_D=1s$. The berth length of the serial station is set to $L=4.4m$ while the distance between successive back-out berth is $L=4.2m$. The other parameter for the back-out stations are $T_{EN}=7s$ and $T_{BO}=12s$. The resulting dwell time ($=T_S/2$) during which passengers can exit and board at a berth of a back-out station is shown in Fig. 5.

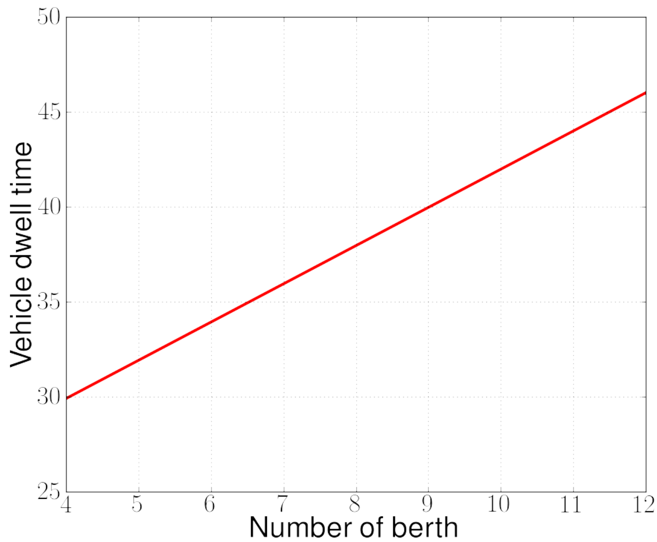


Fig. 5: Dwell time at back-out stations in synchronous operation as a function of berth number $M$.

The station capacities $C_S$ have been determined for berth numbers $M$ from 4 to 12, see Fig. 6. The serial station capacity of type A has been calculated for the four user types from Fig. 2. The capacity of the back-out station is independent of passenger boarding times, as long as they remain below the dwell times shown in Fig. 5. At least for the above system parameters it appears that serial stations do have higher capacities than back-out stations, unless the passengers are extreme slow boarders. However, the above capacities represent average values. In some applications back-out stations do have advantages as single passengers with boarding problems will not block the entire station.

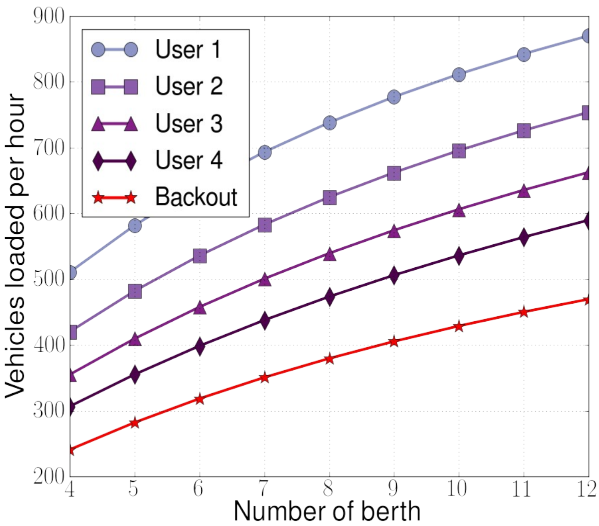


Fig. 6: Station capacities in vehicle loads per hour (with 100 vehicle per hour unloaded) in function of the number of berth. The first 4 curves are the capacities of serial station type A (exit and boarding at same berth) for the different user types from Fig. 2. The last curve is the station capacity of the back-out berth.

The capacities of serial station type A for the four user types from Fig. 2 are shown in Fig. 7. Comparing Fig. 7 with Fig. 6 suggests that separating the unloading and loading in separate zones improves capacity. However, a price to be payed is that the unloading zone requires additional berth in the buffer zone which leads to a higher space requirements and costs.

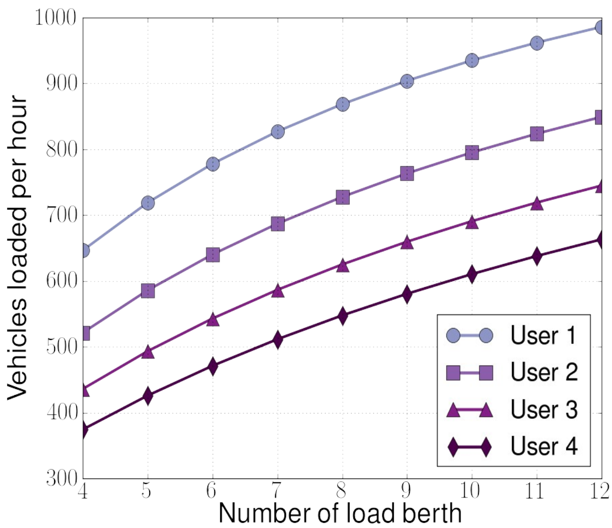


Fig. 7: Station capacities in vehicle loads per hour (with 100 vehicle per hour unloaded) in function of the number of berth. The 4 curves are the capacities of serial station type B (exit and boarding at different berth, see Fig. 1(b).

## 3 Bottlenecks, empty vehicle shares and static vehicle flow assignments

In a first attempt to identify the capacity bottlenecks of a PRT network one can determine full and empty vehicle-flows on each network link by making a static traffic assignment based on a specific network travel demand. This allows to verified whether there is a link-flow in the network that exceeds line capacity. The static flow assignment is not only useful to identify bottlenecks, it can also be used as part of an iterative algorithm to optimize the network design by redirecting links, see Caprara et.al (2008). Below, we briefly explain the PRT assignment method.

The PRT static vehicle flow assignment method is based on linear programming models. In the first place it is necessary to make some basic assumptions:

- all vehicles follow the shortest path from origin to destination. This assumption reflects current PRT control strategies and is equivalent to the classical all-or-nothing assignment (AON).
- the full-vehicle flow must be assigned such that a given demand between each origin-destination pair of the network is satisfied;
- the full-vehicle flow must be counterbalanced by an empty vehicle flow.

For the latter, we add in each station vehicle flow-conservation constraints by defining a fictitious demand of empty vehicles for each station. This fictitious demand represents the algebraic difference between the number of exiting and entering full vehicles at each demand-node. The total demand is the sum of full-vehicle demand and the aforementioned fictitious demand. Then we ensure the flow-equilibrium by adding a multi-origin/multi-destination empty-vehicle flow to the model. This summing of empty and full vehicles is a particular characteristic of PRT networks and has not been addressed sufficiently in literature.

The AON assignment for PRT networks, as proposed in E. Traversi (2009), can be defined by the following LP model. Let $D = (V, A)$ be a directed network graph, where $V$ and $A$ are the nodes and the links of the PRT network, respectively. Each link $a = (i, j) \in A$ is associated with a length (or generalized cost $l_a$). Let $R \subseteq V \times V$ be a set of routes, represented by pairs of nodes and a demand $d_r$ associated with each of the routes $r = (s_r, t_r) \in R$. Let further $S \subset V$ be the set of nodes used at least in one route as origin or destination. Let $s_r$ and $t_r$ be respectively the origin and destination nodes associated with each route $r$. We now define $D_{i,res}$ as the residual demand of node $i$ defined as follows:

$$D_{i,res} = \sum_{r \in R: s_r = i} d_r - \sum_{r \in R: t_r = i} d_r, \quad \forall i \in V \tag{7}$$

where $D_{i,res} > 0$ ($D_{i,res} < 0$) means that there is a demand (offer) of vehicles in node $i$. Let us further introduce the variable $y_{r,a}$, representing the fractional part of the flow on route $r$ using link $a$. The total link-flow is therefore $y_{r,a}$ multiplied by the correspondent demand $d_r$. Another variable is $w_a$, representing the empty vehicle flow on link $a$. The objective function used in the following linear programming (LP) model represents the total distance (or costs) travelled by all full and empty vehicles:

$$\min \sum_{r \in R} \sum_{a \in A} d_r l_a y_{r,a} + \sum_{a \in A} l_a w_a \text{ subject to ...} \tag{8}$$

$$\sum_{a \in \delta^+(i)} y_{r,a} - \sum_{a \in \delta^-(i)} y_{r,a} = \begin{cases} 1 & if \quad i = s_r \\ -1 & if \quad i = t_r \\ 0 & otherwise \end{cases}, \quad \forall i \in V, \quad \forall r \in R \tag{9}$$

$$\sum_{a \in \delta+(i)} w_a - \sum_{a \in \delta-(i)} w_a = D_{i,res}, \quad \forall i \in S \tag{10}$$

$$y_{r,a} \geq 0, \quad \forall a \in A, \quad \forall r \in R \tag{11}$$

$$w_a \geq 0, \quad \forall a \in A \tag{12}$$

The constraints in Eq. 9 guarantee flow conservation for each route in each node and Eq. 10 represents the flow conservation for empty vehicles. Eq. 11 and Eq. 12 ensure all variables are non-negative. For the resolution of the above LP model, which means finding all $y_{r,a}$ and $w_a$ as to minimize Eq. 12, there are well established methods, see Magnanti, Mireault and Wong, 1986. Finally, from the partial path flows $y_{r,a}$ and empty vehicle link flows $w_a$ we can reconstruct the total flow of link $a$ :

$$f_a = \sum_{r \in R} d_r y_{r,a} + w_a \tag{13}$$

Figure 8 illustrates assigned vehicle flows on a section of a larger PRT network. It clearly shows full- and empty-vehicle flows and the capacity limit on each guideway segment.

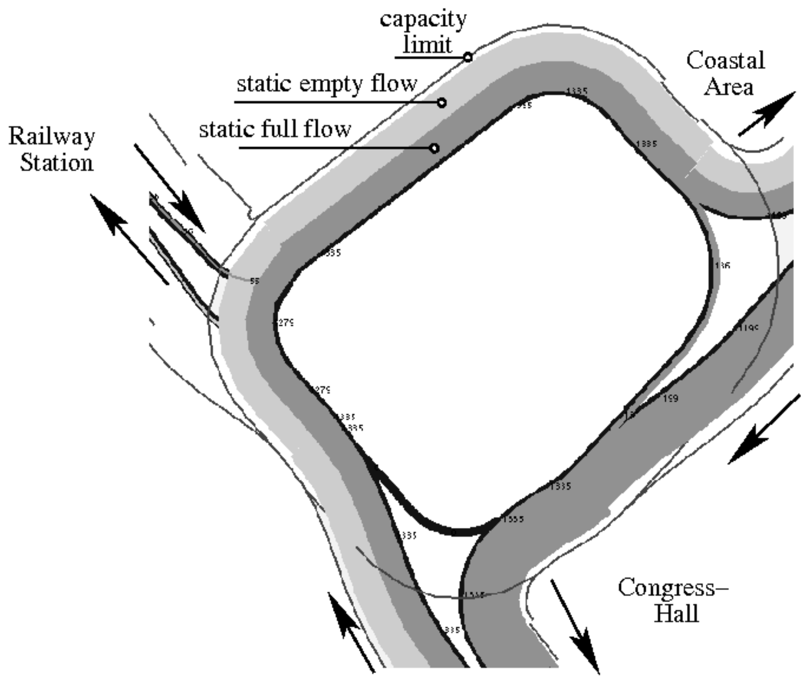


Fig. 8: Central roundabout of the Rimini network, see Traversi (2009), with full vehicle flow, empty vehicle flow and capacity limits.

Note that the flows found by means of the static assignment represent the ideal vehicle flows on the network in the sense that the real flows are likely to be higher. There are two main reasons: (i) the static assignment assumes constant passenger arrival rates at stations during the observation period – in reality there will be variations and the flows will be temporarily higher, even though average flows will match the static flows; (ii) the empty vehicle management of a real PRT system can only react to passengers who have already arrived at stations. This will result in a sub-optimum empty vehicle routing with respect to the static assignment where all origins and destinations are a priori known to the assignment algorithm. This is why the vehicle flows produced by the static assignment can be considered a lower bound for what can be expected in reality. However, the AON approach for full and empty vehicles is not the best routing strategy if capacity constraints are critical. An alternative approach would be to penalize empty vehicles in congested parts of the network, thus forcing them to run along sub-optimal routes; or the link-costs could be increased as the the vehicle flow increases. These ideas would lead to a system optimal traffic assignment strategy which is subject to current research works.

## 4 Conclusions

The capacity analyses of PRT is very spacial due to its high system-complexity between local and global interactions. In order to produce useful results for PRT planners and developers, it has been of particular interest to find analytical relations between design-parameters and capacity limits. As an important example of local interaction, we have addressed the analyses of different station layouts, such as serial and back-out stations. We have further described an assignment algorithm that allows a network-wide flow analyses which is useful to identify capacity bottlenecks. However, the proposed assignment method requires the origin-to-destination demand matrix which is often difficult to estimate. The presented results are not only a useful design-tool but constitute performance bounds which allow to verify microsimulations, carried out in a successive design-step.

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